

QCD factorization at twist-3: the two parton contributions

Tsung-Wen Yeh*

*Department of Science Application And Dissemination,
National Taichung University, Taichung 403, Taiwan*

Abstract

In this paper, the twist-3 two parton corrections in charmless $B \rightarrow PP$ decays are shown to be factorizable under the QCD factorization approach. The factorizability of the twist-3 two parton corrections is constructed on the following findings. Under the energetic meson limit, the pseudoscalar distribution amplitude for a light pseudoscalar meson is allowed to be non-constant by the equations of motion for the quark. The non-constant pseudoscalar distribution amplitude is then used to regularize the end-point divergences in the hard spectator corrections at twist-3 order. By retaining the momentum fraction variable of the spectator quark of the B meson in the propagators, the end-point divergence in the weak annihilation corrections at twist-3 order is resolved. The factorization of the $O(\alpha_s)$ corrections under the two parton approximation is shown valid up-to $O(1/m_b)$. The hard scattering kernels of order $O(\alpha_s)$ and $O(\Lambda_{QCD}/m_b)$ are explicitly given and found to be infrared finite. The results are applied for making predictions for the branching ratios of $B \rightarrow \pi K$ decays.

PACS numbers: 13.25.Hw

Keywords: QCD;Factorization theorem;B decays

*Electronic address: twyeh@ms3.ntcu.edu.tw

I. INTRODUCTION

The hadronic B decays are good places for testing our understanding of standard model (SM) and/or new physics (NP). Until the present time, the B factories have obtained a lot of remarkable results. For example, the $\sin(2\beta)$ has been determined precisely from the measurements of the mixing-induced CP-asymmetry in the $B \rightarrow J/\psi K_s$ decay [1, 2, 3, 4], the branching ratios for $B \rightarrow \pi\pi, \pi K$ decays have been measured with only few percent errors [5, 6, 7], the CP asymmetry for $B \rightarrow \pi^+ K^-$ has been measured precisely [8, 9, 10], and the $B \rightarrow \pi^0 \pi^0$ decay mode has been confirmed experimentally with unexpectedly large branching ratio [6, 11, 12, 13]. In contrast, the theory suffers from large uncertainties from the nonperturbative dynamics of QCD, which are involved in the matrix elements contained in the decay amplitudes. If the theoretical uncertainties can be largely reduced by effective methods, then a deterministic demonstration on the CP mechanism, an optimal extraction of CKM parameters from the wealth of experimental data, and a clean separation of NP from SM could be derived from the investigations of hadronic B decays [14].

The major breakthrough in reducing the theoretical uncertainties comes from an observation [15, 16], which is based on a two loop analysis [17], that the decay amplitudes for nonleptonic B decays can be factorized under the heavy quark mass infinity limit [15, 16, 17, 18, 19]. The factorization of the decay amplitudes means that the amplitudes can be separated into perturbatively calculable short distance functions and nonperturbatively incalculable long distance functions. The finding leads to the QCD factorization (QCDF) [15, 17, 18]. Similar facts have also been found under other approaches, the soft collinear effective theory (SCET) [26, 27, 28, 29, 30, 31, 32, 33] and the perturbative QCD (PQCD) factorization [34, 35, 36, 37, 38, 39, 40, 41, 42].

The important ingredient of QCDF is the factorization theorem [43], which has been one successful method in the studies of hard scattering processes, such as deep inelastic scattering, Drell-Yan processes, etc. Unlike traditional hard scattering processes involving only one single energy scale, the hadronic B decays involve multiple energy scales: the electroweak scale $\mu_{EW} \sim M_W$, the hard scale $\mu_H \sim m_b$, the hard collinear scale $\mu_{HC} \sim \sqrt{m_b \Lambda_{QCD}}$, and the soft scale $\mu_S \sim \Lambda_{QCD}$. This multi-scale characteristic feature of hadronic B decays requires more efforts in the applications of factorization theorem.

The factorization theorem for $B \rightarrow M_1 M_2$ processes with $M_{1,2}$ being light mesons has

been shown up-to leading order (LO) in the heavy mass expansion (or in the $1/m_b$ expansion) and next-to-leading order (NLO) in the loop expansion (or in the α_s expansion). The α_s means the strong coupling constant and m_b is the b quark mass. Throughout this paper, the final state mesons in $B \rightarrow M_1 M_2$ processes are restricted to be pseudoscalar light mesons, π and K . The extension of the results derived in this paper to other light mesons can be done similarly. Under QCDF, the matrix element of a four fermion effective operator Q_i at $O(\alpha_s)$ and LO in $1/m_b$ expansion can be expressed as

$$\begin{aligned} \langle M_1 M_2 | Q_i | B \rangle = & F_j^{BM_1}(q^2) \int_0^1 du T_{ij}^I(u) \phi_{M_2}(u) + (M_1 \leftrightarrow M_2) \\ & + \int_0^1 d\xi \int_0^1 du \int_0^1 dv T_i^{II}(u) \phi_B(\xi) \phi_{M_1}(u) \phi_{M_2}(v) \end{aligned} \quad (1)$$

where $F_j^{BM_1}(q^2)$ with $j = +, 0$ are the transition form factors, $T_{ij}^{I,II}$ are the short distance hard scattering functions and $\phi_B, \phi_{M_1}, \phi_{M_2}$ denote the long distance light cone distribution amplitudes (LCDAs) for the external mesons, B, M_1, M_2 , respectively. The T_{ij}^I function contains contributions of hard scale and the T_i^{II} function contains contributions of hard and hard-collinear scales. The contributions below the soft scale are attributed to the meson LCDAs. The contributions between any two energy scales are calculated by means of the renormalization group method. The prediction power of factorization theorem given in Eq. (1) comes from the universality of the LCDAs. Once the LCDAs are determined experimentally in some processes, they can be applied for making predictions for other processes.

However, the leading order predictions of QCDF based on Eq. (1) can not consistently accommodate with experimental data for many decay processes. For example, the theoretical predictions are about one half of the experimental data for the branching ratios of $B \rightarrow \pi K$ decays [20, 21]. For understanding the experiments, high order corrections to the factorization equation Eq. (1) would be considered [44]. The higher order corrections mean higher loop corrections in the α_s expansion or higher twist corrections in the $1/m_b$ expansion. As the higher order corrections are considered within QCDF formalism, it is an important issue to investigate whether the factorization of higher order corrections is still valid.

So far, only partial results of order $O(\alpha_s^2)$ for $T^{I,II}$ have been calculated [45, 46, 47, 48, 49, 50, 51, 52, 53]. These calculations showed that the $O(\alpha_s^2)$ corrections seem preserving the factorization. On the other hand, the factorization of higher twist corrections is still

unclear. It was found that some high twist hard spectator and weak annihilation corrections spoil the factorization. The twist-3 hard spectator corrections [20] contain the divergent term

$$X_H = \int_0^1 \frac{du}{u} \phi_p(u) , \quad (2)$$

where $\phi_p(u)$ is a twist-3 two particle pseudoscalar LCDA for a pseudoscalar light meson. Because $\phi_p(u) = 1$ in the chiral limit [64, 66], X_H diverges at $u = 0$. Similar end-point divergences also happen in the weak annihilation corrections, in which the divergent term is

$$X_A = \int_0^1 \frac{du}{u^2} \phi_P(u) , \quad (3)$$

where $\phi_P(u)$ is a twist-2 LCDA for a pseudoscalar light meson. These end-point divergences spoil the QCD factorization at the order of $O(\alpha_s)$ and $O(1/m_b)$. Since the X_H term are related to chirally enhanced corrections, which have a numerically large factor $r_\chi \sim O(1)$, the power corrections can be equally important as the radiative corrections in charmless hadronic B decays [20]. From this respect, the extension of QCDF to subleading twist order is of some urgency [14, 20].

However, the systematic generalization of QCDF to subleading twist order is still not available. For this reason, we restrict ourselves in this paper to only investigate the physics related to the non-factorizability of the $X_{H,A}$ and search for a possible resolution to the related end-point divergences. In QCDF, the factorization is based on the collinear factorization scheme in which the partons participating in the hard scattering kernels are assumed to carry only collinear momenta. The hard scattering kernels are calculated by means of a leading twist collinear expansion method and perturbative QCD. The models for the LCDAs are submitted to nonperturbative theories, such as the QCD sum rules. Since the $X_{H,A}$ terms are related to twist-3 contributions for $B \rightarrow M_1 M_2$ decays, the calculation scheme for the leading twist hard scattering kernels needs to be generalized. We will employ the collinear expansion (CE) proposed by Yeh [57] to calculate the relevant hard scattering functions up-to twist-3. The CE calculation scheme is a generalization of the leading twist collinear expansion for hard scattering processes [54, 55, 56, 59, 60, 61, 62, 63]. Similar extensions of the leading twist collinear expansion scheme for only two parton contributions have been proposed by Beneke and Neubert (the BN scheme) [20] and by Du, Yang and Zhu (the DYZ scheme) [23, 24].

Not only the calculation scheme needs to be generalized, but also the twist-3 LCDAs for the light mesons are required to be defined consistently to derive a factorization theorem at twist-3 order for $B \rightarrow M_1 M_2$ decays. For the X_H term and other twist-3 two parton contributions for $B \rightarrow M_1 M_2$ decays, two twist-3 two particle LCDAs $\phi_p(u)$ and $\phi_\sigma(u)$ are involved. For a pseudoscalar light meson, there are three twist-3 LCDAs, the pseudoscalar LCDA $\phi_p(u)$, the pseudotensor LCDA $\phi_\sigma(u)$, and the three particle LCDA $\phi_3(u, u')$. These three twist-3 LCDAs are related to each other by equations of motion (EOM)[64, 66]. Due to its small normalization factor, the three particle LCDA $\phi_3(u, u')$ is usually neglected in literature. However, this may not be a good approximation for some processes. For example, in the penguin dominated B decays, the tree level contributions from the three particle LCDA could be as large as the tree level contributions from the other two particle LCDAs [57].

In the approximation of neglecting the three particle contributions, $\phi_p(u)$ and $\phi_\sigma(u)$ are determined by the following equations

$$\begin{aligned} \frac{\bar{u}}{2}(\phi_p(u) - \frac{1}{6} \frac{d\phi_\sigma(u)}{du}) &= \frac{1}{6} \phi_\sigma(u) , \\ \frac{u}{2}(\phi_p(u) + \frac{1}{6} \frac{d\phi_\sigma(u)}{du}) &= \frac{1}{6} \phi_\sigma(u) , \end{aligned} \quad (4)$$

where $\bar{u} = 1 - u$. The solutions are: $\phi_\sigma(u) = 6u\bar{u}$ and $\phi_p(u) = 1$. We denote the solutions as the chiral limits of the $\phi_p(u)$ and $\phi_\sigma(u)$.

As explained above, the substitution of $\phi_p(u) = 1$ into the X_H term results in an end-point divergence at $u = 0$. This end-point divergence may be due to the failure of the collinear factorization scheme, or the incorrect use of a model for the $\phi_p(u)$. In this paper, we intend to assume that the collinear factorization scheme is still applicable for the X_H term and to study a consistent model for the $\phi_p(u)$. This is different from the common viewpoint for this divergent problem as taken in the literature. In this respect, we found that the energetic meson limit is an important condition for solving the divergent problem in the $X_{H,A}$ term. The energetic meson limit is defined as the limit at which the light meson's momentum becomes energetic. The energetic momentum means that the momentum contains a large component which is much larger than the light meson's mass and other components of the momentum. One light meson carries an energetic momentum is defined as an energetic meson. The energetic meson limit is the condition under which the leading twist factorization theorem Eq. (1) has been shown to be valid. For example, the $O(\alpha_s)$ radiative corrections

for the Feynman diagrams as depicted in Fig. 1(a)-(d) can only be shown infrared finite by requiring the emitted meson M_2 to carry an energetic momentum such that the partons inside the M_2 meson can have large collinear momenta. The collinear (infrared) divergences cancel out in pairs between these four diagrams under the condition that the external partons to the radiative loops are collinear to their parent meson M_2 [17, 18]. Because no any evidence shows that the higher twist contributions require different kinematics, we argue that, similar to the leading twist contributions, the twist-3 contributions (including the $X_{H,A}$) should be derived by using the same energetic meson limit, too. This argument has been used to derive the X_H term in [20]. Therefore, if the energetic meson limits for $\phi_p(u)$ and $\phi_\sigma(u)$ are different from the chiral limits, then one can hope to find a resolution to the divergent problem in X_H .

However, the energetic meson limits for $\phi_p(u)$ and $\phi_\sigma(u)$ have not been studied in literature. We need to derive them in this paper. The details for derivations of the energetic meson limits for $\phi_p(u)$ and $\phi_\sigma(u)$ will be given in Section II. Here, we briefly describe how the energetic meson limits for $\phi_p(u)$ and $\phi_\sigma(u)$ can be different from the chiral limits. If the pseudoscalar light meson is energetic, then the $\phi_\sigma(u)$ at $u = O(1)$ is found to be of $O(\Lambda/E)$. The $\phi'_\sigma(u) = O(1)$ (the derivative of $\phi_\sigma(u)$) and the $\phi_p(u) = O(1)$ at $u = O(1)$. The E denotes the energy of the energetic meson and Λ is of $O(\Lambda_{QCD})$. It implies that the $\phi_\sigma(u)$ and the $\phi'_\sigma(u)$ are of different order under the energetic meson limit, and they should be defined as different LCDAs. To keep both sides of Eq. (4) of the same order, the $\phi_\sigma(u)$ should be dropped out and Eq. (4) is further reduced to contain only the $\phi_p(u)$ and the $\phi'_\sigma(u)$. The factor $1/6$ associated with $\phi'_\sigma(u)$ in the Eq. (4) is only for normalization, it is then instructive to redefine the pseudotensor LCDA as that $\hat{\phi}_\sigma(u)$ defined in Eq. (23). By using the $\hat{\phi}_\sigma(u)$, the appropriate EOM for $\phi_p(u)$ in the energetic meson limit becomes

$$\hat{\phi}_p(u) = \hat{\phi}_\sigma(u) , \quad (5)$$

where the $\hat{\phi}_p(u)$ and $\hat{\phi}_\sigma(u)$ are defined as the energetic meson limits of $\phi_p(u)$ and $\phi_\sigma(u)$, respectively.

To have a better understanding of the above fact, we can define $\Delta\phi_\sigma(u) = (\phi_\sigma(u) - \hat{\phi}_\sigma(u))$ as the difference between $\phi_\sigma(u)$ and $\hat{\phi}_\sigma(u)$ by referring to Eq. (8) and Eq. (23). The $\phi_\sigma(u)$ in Eq. (4) should be $\Delta\phi_\sigma(u)$, and $\Delta\phi_\sigma(u)$ is of $O(\Lambda/E)$ in comparison to $\phi'_\sigma(u)$ at $u = O(1)$. and should be identified as a twist-4 quantity in the energetic meson limit $E \gg \Lambda$.

Another interpretation is as following. According to Eq. (8), it is better to expand the coordinate variable z_β in the spin projector for the $\phi_\sigma(u)$ into its collinear and transverse parts, and to assign them by corresponding LCDAs, denoted by $\phi'_\sigma(u)$ and $\phi_\sigma(u)$, respectively. To have a more transparent notation, we define $\phi_\sigma^\parallel(u) \equiv \phi'_\sigma(u)$ and $\phi_\sigma^\perp(u) \equiv \phi_\sigma(u)$, which respect the collinear degrees of freedom and the transverse degrees of freedom of the light meson state $|M\rangle$, respectively. These two $\phi_\sigma^\parallel(u)$ and $\phi_\sigma^\perp(u)$ are equally important at the condition $E \simeq \Lambda$, which is applicable for a soft light meson. However, $\phi_\sigma^\parallel(u)$ and $\phi_\sigma^\perp(u)$ become of different order as we boost the reference frame along the collinear direction of the light meson's momentum such that $E \gg \Lambda$, which is applicable for an energetic light meson. In the $E \gg \Lambda$ reference frame, $\phi_\sigma^\perp(u)$ becomes suppressed by a factor Λ/E than $\phi_\sigma^\parallel(u)$. This is consistent with the parton model picture that the collinear degrees of freedom dominate over the transverse ones. In summary, we arrive at two consistent explanations for the same thing by noting that $\phi_\sigma^\parallel(u) \propto \hat{\phi}_\sigma(u)$ and $\phi_\sigma^\perp(u) \propto \Delta\phi_\sigma(u)$.

To clearly clarify the source for the divergence in the X_H term, we also need to examine the calculation method related to the $\phi_\sigma(u)$. The method proposed by Beneke and Neubert [20] is to separate the spin projector for the $\phi_\sigma(u)$ into the collinear and transverse parts (the BN scheme). The collinear part is transformed into a derivative over the momentum fraction u and the (collinear) derivative is then defined to be applied on the $\phi_\sigma(u)$. The transverse part is transformed into a momentum derivative in the transverse direction, and the (transverse) derivative is then defined to be applied on the hard scattering kernel. Alternatively, one can also let the collinear derivative applied on the hard scattering kernel and the transverse derivative applied on the $\phi_\sigma(u)$. These two approaches are equivalent mathematically, but may result in different physical results. It is known as the projection ambiguity. To solve this ambiguity, Du, Yang and Zhu (DYZ) [23, 24] proposed that the whole momentum derivative should be applied on the hard scattering kernel (the DYZ scheme). As mentioned previously, the $\phi_\sigma^\parallel(u)$ and the $\phi_\sigma^\perp(u)$ are of different magnitudes in the energetic meson limit, and their associated hard scattering kernels should be calculated separately. Since the above two calculation schemes did not consider the difference between $\phi_\sigma^\parallel(u)$ and $\phi_\sigma^\perp(u)$, the results calculated by these two schemes require further examinations. In this paper, we propose to employ the collinear expansion method to re-calculate the twist-3 two parton contributions. The collinear expansion method will be described in details in Section III.

For the weak non-singlet annihilation corrections, the end-point divergent problem is

more severe because it exists even for leading twist-2 LCDAs. This can be seen from the X_A term in Eq. (3). To solve this problem, we propose to retain the momentum fraction variable of the spectator quark of the B meson in the denominators of the parton propagators. In literature, the momentum fraction variable of the spectator quark of the B meson is always neglected. This is because the distribution function for the B meson is highly asymmetric such that the momentum fraction variable of the spectator quark of the B meson is of order Λ/m_b . The reason to retain the momentum fraction variable of the spectator quark of the B meson is as following. The divergence in X_A term mainly arises as the parton propagators of the B meson become on-shell. To regularize the divergence in X_A term, we let the propagator be slightly off-shell by adding a term of $O(\Lambda^2/m_b^2)$ into it. After applying this for calculation, there are two same factors in the numerator and the denominator of the propagator of the spectator particle, respectively. As a result, these two factors cancel. The X_A in Eq. (3) becomes

$$X_A \rightarrow \int_0^1 d\xi \phi_B(\xi) \int_0^1 \frac{du}{(\bar{u} - \xi)\bar{u}} \phi_P(u) . \quad (6)$$

It is obvious that the original end-point divergence is regularized by the momentum fraction ξ carried by the spectator particle. The only price we need to pay is to retain the $\phi_B(\xi)$ without integrating it out. From the resultant expression, the spectator particle is interpreted to carry a collinear momentum, although the spectator particle's momentum is soft. The above argument is valid for $(V - A)(V \pm A)$ and $-2(S - P)(S + P)$ operators. Therefore, the divergent problem associated with the X_A term is resolved. Similar fact for the factorizability of the annihilation contributions has been observed in [67, 68]

The organization of this paper is as follows. In Section II, the energetic meson limits for $\phi_p(u)$ and $\phi_\sigma(u)$ and related physics will be studied in details. In Section III, a simple introduction to the CE expansion scheme will be given first. The CE scheme is then applied to analyze the next-to-leading order radiative corrections. The factorization for the next-to-leading order radiative corrections is shown to be valid at $O(1/E)$ or $O(1/m_b)$. In Section IV, the CE scheme is compared to the BN and DYZ schemes for the contributions related to the $\phi_\sigma(u)$. In Section V, the decay amplitudes at twist-3 order and at α_s order will be recalculated under the CE scheme. The explicit expressions for the amplitudes for $B \rightarrow PP$ decays, in which the final state PP means pseudoscalar light mesons, will be given in this Section, too. The predictions for the branching ratios of $B \rightarrow \pi K$ decays will be also present.

The last Section is devoted for conclusions. The calculation details for the twist-3 $O(\alpha_s)$ vertex and penguin corrections are given in Appendix A and B.

II. THE ENERGETIC MESON LIMIT AND THE LIGHT CONE DISTRIBUTION AMPLITUDES FOR LIGHT PSEUDOSCALAR MESONS

In the $B \rightarrow M_1 M_2$ decays with $M_{1,2}$ being light mesons, the momenta P_1 and P_2 of the M_1 and M_2 mesons can have a component being much larger than the other components and the meson mass. In the rest mass frame of the decaying B meson, the momentum conservation $P_B = P_1 + P_2$ and the smallness of the meson masses $m_{M_1}, m_{M_2} \ll M_B$ lead to $P_1 \cdot P_2 = (M_B^2 - m_{M_1}^2 - m_{M_2}^2)/2 \simeq M_B^2/2$ and $P_1^\mu = (P_1^+, P_2^-, \vec{P}_{1\perp}) = (M_B/\sqrt{2}, 0, \vec{0})$ and $P_2^\mu = (P_2^+, P_2^-, \vec{P}_{2\perp}) = (0, M_B/\sqrt{2}, \vec{0})$. The light mesons in this situation are identified as energetic mesons. The energetic mesons are not limited in the charmless hadronic B decays. The light mesons in the hadronic $B \rightarrow DM$ decays with heavy-light final state mesons, or the exclusive hard scattering processes, e.g., $\pi\gamma^* \rightarrow \gamma$, $\pi\gamma^* \rightarrow \pi$, can all be considered as energetic mesons. For convenience, we parameterize the momentum P^μ of an energetic meson as

$$P^\mu = E\bar{n}^\mu + \frac{m_M^2}{2E}n^\mu, \quad (7)$$

in which $E = M_B/\sqrt{2}$ and two light-like vectors $\bar{n}^\mu = (\bar{n}^+, \bar{n}^-, \vec{\bar{n}}_\perp) = (1, 0, \vec{0}_\perp)$ and $n^\mu = (n^+, n^-, \vec{n}_\perp) = (0, 1, \vec{0}_\perp)$ are introduced. The vectors n^μ and \bar{n}^μ satisfy $\bar{n}^2 = n^2 = 0$ and $\bar{n} \cdot n = 1$. For later discussions, we define the chiral limit as $E \sim m_M \sim \Lambda_{QCD}$, and the energetic meson limit as $E \gg m_M, \Lambda_{QCD}$.

The meson distribution amplitudes with a given twist order are defined as the momentum fraction distributions of partons in a particular Fock state of a meson. Up to twist-3, the distribution amplitudes of a pseudoscalar meson M are defined by the matrix element of nonlocal operators [64]

$$\begin{aligned} & \langle M(P) | \bar{q}(0)[0; z]q(z) | 0 \rangle|_{z^2=0} \\ &= -i\frac{f_M}{4} \int_0^1 du e^{i\bar{u}P \cdot z} [\gamma_5 \gamma^\mu P_\mu \phi_P(u) + \mu_\chi^M \gamma_5 \left(\phi_p(u) - \sigma^{\mu\nu} P_\mu z_\nu \frac{\phi_\sigma(u)}{6} \right)] \end{aligned} \quad (8)$$

where $[0; z]$ denotes the Wilson line for preserving the gauge invariance of the matrix element. P is the meson momentum, f_M the decay constant of the meson M , and $\mu_\chi^M = m_M^2/(m_q +$

$m_{\bar{q}}$) with m_M , m_q and $m_{\bar{q}}$ being the meson's mass and the quark and anti-quark current masses. The twist-2 distribution amplitude $\phi_P(u)$ and the twist-3 two particle distribution amplitudes $\phi_p(u)$ and $\phi_\sigma(u)$ have been studied, in the chiral limit, based on nonlocal product expansion and conformal expansion. In addition, the equations of motion of on-shell quarks in the meson were used to obtain two differential-integral relations between the twist-3 two particle distribution amplitudes $\phi_p(u)$ and $\phi_\sigma(u)$ and the twist-3 three particle distribution amplitude $\phi_3(u, u')$. The differential-integral relations can be solved by means of moment. $\phi_p(u)$ and $\phi_\sigma(u)$ are determined by $\phi_3(u, u')$ [64].

Some simplifications can be obtained by neglecting the twist-3 three particle distribution amplitude $\phi_3(u, u')$ due to its small normalization constant. The two differential-integral relations are then reduced to the relations as shown in Eq. (4). We identify the solutions to Eq. (4) as the chiral limits $\phi_p^c(u)$ and $\phi_\sigma^c(u)$ of $\phi_p(u)$ and $\phi_\sigma(u)$, in which $\phi_p^c(u) = 1$ and $\phi_\sigma^c(u) = 6u\bar{u}$ with $\bar{u} = 1 - u$.

Referring to Eq. (2), the X_H term from the hard spectator diagrams contains the partonic part $1/u$ and the hadronic part $\phi_p(u)$. As the chiral limit solution $\phi_p^c(u)$ is substituted into the hadronic part $\phi_p(u)$ of the X_H term, an end-point divergence arises as $u \rightarrow 0$. The end-point divergence spoils the factorization at the twist-3 order for $B \rightarrow M_1 M_2$ processes. In literature, the common viewpoint is to identify that the breakdown of the factorization for the X_H term is due to the failure of the factorization scheme, the QCD factorization. In this paper, we propose to take another viewpoint that the source for the breakdown of the factorization for the X_H term may be due to the use of the chiral limit solution for the $\phi_p(u)$. Our consideration is the following. The partonic part of the X_H term is derived under the assumption that the external meson is taken in its energetic limit [20], while the hadronic part is using the chiral limit solution for the relevant distribution amplitude. If the energetic limit solution for $\phi_p(u)$ can be different from the chiral limit solution, then we can expect to find a resolution for the end-point divergence.

To see whether the above argument is correct, it is necessary to find out the energetic limits, $\hat{\phi}_p(u)$ and $\hat{\phi}_\sigma(u)$, of the $\phi_p(u)$ and $\phi_\sigma(u)$. Since the energetic limits have not been studied, we will derive them in this paper. The first main result of this paper is to show that the energetic limits, $\hat{\phi}_p(u)$ and $\hat{\phi}_\sigma(u)$, indeed exist and are different from the chiral limits. We will employ a simplified method to derive similar relations to those in Eq. (4) for $\hat{\phi}_p(u)$ and $\hat{\phi}_\sigma(u)$. According to the relations for $\hat{\phi}_p(u)$ and $\hat{\phi}_\sigma(u)$, a non-constant solution

for the $\hat{\phi}_p(u)$ will be obtained. We will also develop an expansion scheme consistent with assumption for taking the energetic limit for the light mesons. The expansion scheme is the second main result of this paper and will be given in Section III. By using the expansion scheme to re-derive the X_H term and substituting the non-constant solution for the $\hat{\phi}_p(u)$ into the hadronic part of the X_H term, the end-point divergence is then resolved.

In the following derivation, only the asymptotic solutions for $\hat{\phi}_p(u)$ and $\hat{\phi}_\sigma(u)$ are considered. The complete solutions for the $\hat{\phi}_p(u)$ and $\hat{\phi}_\sigma(u)$ by following the traditional approaches [64, 65, 66] will be given in another place. Usually, the twist-2 LCDA $\phi_P(u)$ is defined as the probability of the transition of the meson M into the $q(u)\bar{q}(1-u)$ pair at zero transverse distance. We assume that the same conditions are also applicable for the $\hat{\phi}_p(u)$ and $\hat{\phi}_\sigma(u)$. To define the $\hat{\phi}_p(u)$ and $\hat{\phi}_\sigma(u)$, we start from Eq. (8). For convenience, the coordinate z in the matrix element can be parameterized under the energetic limit as the following

$$z_\beta = \frac{z_\perp^2 E}{2\lambda} \bar{n}_\beta + \frac{\lambda}{E} n_\beta + z_{\perp\beta} , \quad (9)$$

where the variable λ is for boost invariance in the collinear direction and z_\perp is assumed to be of order $O(1/E)$. It is noted that the λ is required to be large to insure that the collinear component $\bar{z}^\mu \equiv \lambda/E n^\mu$ dominates. This requirement for λ is consistent with the condition for an energetic meson in the highly boost frame. The quark field $q(z)$ in Eq. (8) can be expanded with respect to $\bar{z}^\mu \equiv \lambda/E n^\mu$ as

$$q(z) = q(\bar{z}) + \left. \frac{\partial q(z)}{\partial z^\mu} \right|_{z=\bar{z}} (z - \bar{z})^\mu + \dots , \quad (10)$$

where dots means terms of order $O(z_\perp^n)$ with $n \geq 2$. By using Eqs. (7) and (9), the spin projector $[P_\alpha, z_\beta]$ associated with $\phi_\sigma(u)$ defined in Eq. (8) can be written as

$$\begin{aligned} [P_\alpha, z_\beta] &= \lambda[\bar{n}_\alpha, n_\beta] - \frac{m_M^2 z_\perp^2}{4\lambda} [\bar{n}_\alpha, n_\beta] \\ &\quad + E[\bar{n}_\alpha, z_{\perp\beta}] + \frac{m_M^2}{2E} [n_\alpha, z_{\perp\beta}] . \end{aligned} \quad (11)$$

By substituting Eqs. (10) and (11), into Eq. (8), we arrive at the following identity up to $O(z_\perp)$

$$\begin{aligned} &\langle M(P) | \bar{q}(0) \sigma_{\alpha\beta} \gamma_5 q(\lambda/E n) | 0 \rangle + \left. \langle M(P) | \bar{q}(0) \sigma_{\alpha\beta} \gamma_5 \frac{\partial q(z)}{\partial z^\mu} | 0 \rangle \right|_{z=\bar{z}} z_\perp^\mu \\ &= -i f_M \mu_\chi^M \left\{ -i[\bar{n}_\alpha, n_\beta] \int_0^1 du e^{i\bar{u}\lambda} \frac{1}{6} \frac{d\phi_\sigma(u)}{du} + E[\bar{n}_\alpha, z_{\perp\beta}] \int_0^1 du e^{i\bar{u}\lambda} \frac{\phi_\sigma(u)}{6} \right\} \end{aligned}$$

$$+i [\bar{n}_\alpha, n_\beta] \left(e^{i\bar{u}\lambda} \frac{\phi_\sigma(u)}{6} \right)_{u=0}^{u=1} \Bigg\} , \quad (12)$$

where the $P \cdot z$ in the phase factor $\exp(i\bar{u}P \cdot z)$ has been approximated to be λ , and the terms proportional to m_M or z_\perp^2 have been neglected. An integration by parts has been used to obtain the first and third terms in the right hand side of Eq. (12). According to the convention of [18, 20], we let $\phi'_\sigma(u)$ and $\phi_\sigma(u)$ correspond to the collinear part and the transverse part of the spin projector, respectively. Comparing both sides of Eq. (12), we choose the following identities according to the order of the z_\perp factor

$$\langle M(P) | \bar{q}(0) \sigma_{\alpha\beta} \gamma_5 q(\lambda/En) | 0 \rangle = -f_M \mu_\chi^M [\bar{n}_\alpha, n_\beta] \frac{1}{6} \left\{ \int_0^1 du e^{i\bar{u}\lambda} \frac{d\phi_\sigma(u)}{du} - \left(e^{i\bar{u}\lambda} \phi_\sigma(u) \right)_{u=0}^{u=1} \right\} , \quad (13)$$

$$\langle M(P) | \bar{q}(0) \sigma_{\alpha\beta} \gamma_5 \partial_\mu q(\lambda/En) | 0 \rangle z_\perp^\mu = -i f_M \mu_\chi^M E [\bar{n}_\alpha, z_\perp] \int_0^1 du e^{i\bar{u}\lambda} \frac{\phi_\sigma(u)}{6} . \quad (14)$$

From Eqs. (13), (14) and (8), we observe the following issues needed for further examinations in the energetic limit:

- The meaning of the momentum fraction u : according to the parton model, the momentum fraction u is defined to be the ratio of the collinear parts of the parton's momentum and the meson's momentum. That is, if we let k denote the parton's momentum, then the momentum fraction is defined as $\hat{u} = k^+/P^+$, where we have used the \hat{u} to distinguish from the u . However, the momentum fraction u in Eq. (8) is not defined as the \hat{u} . On the other hand, the u in Eq. (8) can be interpreted as the fraction of the whole momentum P . This can be seen from the $\exp(i\bar{u}P \cdot z)$ in Eq. (8). If the parton momentum is denoted as k , then $u = k \cdot z / P \cdot z$. By using the parameterizations for P and z , the u is expanded as

$$u = \frac{k \cdot z}{P \cdot z} = \frac{\frac{z_\perp^2 k_\perp^2 E}{4\lambda k \cdot n} + k \cdot n \frac{\lambda}{E} - \vec{k}_\perp \cdot \vec{z}_\perp}{\lambda + \frac{m_M^2 z_\perp^2}{4\lambda}} , \quad (15)$$

where the parton momentum k^μ has been parameterized as

$$k^\mu = k \cdot n \bar{n}^\mu + \frac{k_\perp^2}{2k \cdot n} n^\mu + \vec{k}_\perp . \quad (16)$$

The momentum fraction u carried by the collinear partons should be defined under the limit $|z_\perp| \rightarrow 0$, $\hat{u} = \lim_{|z_\perp| \rightarrow 0} u(z_\perp) \equiv k \cdot n / P \cdot n$. The fraction u in Eqs. (13) and (14) should be interpreted as \hat{u} . In the following text, the u is always interpreted as \hat{u} to simplify the notations.

- The boundary condition: the boundary terms in Eq. (13) implies the following equation

$$\left(\phi_\sigma(1) - e^{i\lambda}\phi_\sigma(0)\right) = 0. \quad (17)$$

Although the solution $\phi_\sigma(1) = \phi_\sigma(0) = 0$ can satisfy the above equation, but the end-point $u \rightarrow 1$ behavior of $\phi_\sigma(u)$ (, or, by translation invariance, the end-point $u \rightarrow 0$ behavior of $\phi_\sigma(u)$) is related to λ . Since the λ depends on the reference frame, this is in contradiction with the universality assumption for the $\phi_\sigma(u)$.

- The definition for the $\phi_\sigma(u)$: take a differentiation on both sides of Eq. (14) in z_\perp as

$$\langle M(P) | \bar{q}(0) \sigma_{\alpha\beta} \gamma_5 \partial_\mu (\lambda/En) q(\lambda/En) | 0 \rangle = -i f_M \mu_M E [\bar{n}_\alpha, d_{\perp\mu\beta}] \int_0^1 du e^{i\bar{u}\lambda} \frac{\phi_\sigma(u)}{6} \quad (18)$$

We observe from Eqs. (13) and (18) that the same $\phi_\sigma(u)$ corresponds to two different matrix elements in the energetic limit. There arise confusions which definition, Eq. (13) or Eq. (18) should be used for $\phi_\sigma(u)$.

- The equations of motion: to avoid the above confusions in the definition for $\phi_\sigma(u)$, we suggest to re-define $\phi_\sigma^\parallel(u) = \phi'_\sigma(u)$ and $\phi_\sigma^\perp(u) = \phi_\sigma(u)$ according to Eq. (13) and Eq. (18), respectively. Because the boundary term in Eq. (13) may not vanish, the boundary term is defined to be absorbed by $\phi_\sigma^\parallel(u)$. By using $\phi_\sigma^\parallel(u)$ and $\phi_\sigma^\perp(u)$, the first equation in Eq. (4) can be written as

$$\frac{\bar{u}}{2}(\phi_p(u) - \frac{1}{6}\phi_\sigma^\parallel(u)) = \frac{1}{6}\phi_\sigma^\perp(u). \quad (19)$$

To find out the energetic limit of the above equation, it is instructive to rewrite $\phi_\sigma^\parallel(u)$ and $\phi_\sigma^\perp(u)$ as the following expressions by taking Fourier transformations for Eqs. (13) and (18)

$$\phi_\sigma^\parallel(u) = \frac{1}{f_M \mu_\chi^M} \int_0^\infty \frac{d\lambda}{2\pi} e^{-i\bar{u}\lambda} \langle M(P) | \bar{q}(0) \sigma_{\alpha\beta} \gamma_5 n^\alpha \bar{n}^\beta q(\lambda/En) | 0 \rangle, \quad (20)$$

$$\phi_\sigma^\perp(u) = \frac{i}{f_M \mu_\chi^M} \int_0^\infty \frac{d\lambda}{2\pi} \frac{e^{-i\bar{u}\lambda}}{E} \langle M(P) | \bar{q}(0) \sigma_{\alpha\beta} \gamma_5 n^\alpha \partial_\perp^\beta (\lambda/En) q(\lambda/En) | 0 \rangle. \quad (21)$$

It is seen that, in Eq. (21), there is a large factor $1/E$ for $\phi_\sigma^\perp(u)$, which is of short distance. The transverse derivative ∂_\perp^β in the matrix element $\langle M(P) | \bar{q}(0) \sigma_{\alpha\beta} \gamma_5 n^\alpha \partial_\perp^\beta (\lambda/En) q(\lambda/En) | 0 \rangle$ corresponds to the transverse momentum k_\perp^β for the quarks in the meson, which is of order $O(\Lambda)$. Therefore, $\phi_\sigma^\perp(u)$ is of order

$O(\Lambda/E)$ for $u = O(1)$ in the energetic limit. On the other hand, $\phi_\sigma^\parallel(u)$ and $\phi_p(u)$ are of order $O(1)$ for $u = O(1)$ in the energetic limit. The power counting for $\phi_\sigma^\perp(u)$, $\phi_\sigma^\parallel(u)$ and $\phi_p(u)$ implies that Eq. (19) is reduced to $\phi_p(u) = \phi_\sigma^\parallel(u)/6$ in the energetic limit.

Based on the above discussions, we arrive at the following definitions for the $\hat{\phi}_p(u)$ and $\hat{\phi}_\sigma(u)$

$$\langle M(P) | \bar{q}(0) i \gamma_5 q(\lambda/E n) | 0 \rangle = f_M \mu_\chi^M \int_0^1 du e^{i\bar{u}\lambda} \hat{\phi}_p(u) , \quad (22)$$

$$\langle M(P) | \bar{q}(0) \sigma_{\alpha\beta} \gamma_5 q(\lambda n/E) | 0 \rangle = -f_M \mu_\chi^M [\bar{n}_\alpha, n_\beta] \int_0^1 du e^{i\bar{u}\lambda} \hat{\phi}_\sigma(u) . \quad (23)$$

We now show that the equations of motion for $\phi_p(u)$ and $\phi_\sigma(u)$ in the energetic meson limit is

$$\hat{\phi}_p(u) = \hat{\phi}_\sigma(u) . \quad (24)$$

Let's start from the following equation

$$i f_M \mu_\chi^M (\bar{u} \hat{\phi}_p(u)) = \int_0^\infty \frac{d\lambda}{2\pi} \frac{e^{-i\bar{u}\lambda}}{E} \langle M | \bar{q}(0) \gamma_5 i n \cdot \partial(\lambda n/E) q(\lambda n/E) | 0 \rangle - f_M \mu_\chi^M . \quad (25)$$

To arrive at the above equation, we have assumed the boundary conditions

$$\lim_{\lambda \rightarrow \infty} e^{-i\bar{u}\lambda} \langle M | \bar{q}(0) \gamma_5 q(\lambda n/E) | 0 \rangle = 0 , \lim_{\lambda \rightarrow 0} e^{-i\bar{u}\lambda} \langle M | \bar{q}(0) \gamma_5 q(\lambda n/E) | 0 \rangle = -i f_M \mu_\chi^M , \quad (26)$$

where the first holds due to the large fluctuation in $e^{-i\bar{u}\lambda}$ under $\lambda \rightarrow \infty$, and the second is the normalization condition. By using the identity $n \cdot (i\partial) = \not{n} i \not{\partial} - \sigma_{\alpha\beta} n^\alpha (\partial^\beta)$ and the fact that

$$\int_0^\infty \frac{d\lambda}{2\pi} \frac{e^{-i\bar{u}\lambda}}{E} \langle M | \bar{q}(0) \gamma_5 \sigma_{\alpha\beta} n^\alpha \partial^\beta (\lambda n/E) q(\lambda n/E) | 0 \rangle = -i f_M \mu_\chi^M (\bar{u} \hat{\phi}_\sigma(u)) - f_M \mu_\chi^M ,$$

we then obtain

$$i f_M \mu_M (\bar{u} \hat{\phi}_p(u)) = \int_0^\infty \frac{d\lambda}{2\pi} e^{-i\bar{u}\lambda} \langle M | \bar{q}(0) \gamma_5 \frac{\not{n}}{E} i \not{\partial} (\lambda/E n) q(\lambda n) | 0 \rangle + i f_M \mu_\chi^M (\bar{u} \hat{\phi}_\sigma(u)) , \quad (27)$$

where $\hat{\phi}_\sigma(u)$ has been defined in Eq. (23).

For latter uses, we use the identity

$$\gamma_5 \sigma_{\alpha\beta} = \frac{i}{2} \epsilon_{\alpha\beta\eta\lambda} \sigma^{\eta\lambda} \quad (28)$$

to re-express $\hat{\phi}_\sigma(u)$ in a form as

$$f_M \mu_\chi^M \hat{\phi}_\sigma(u) = -\frac{i}{2} \int_0^\infty \frac{d\lambda}{2\pi} e^{-i\bar{u}\lambda} \langle M | \bar{q}(0) \epsilon_\perp^{\alpha\beta} \sigma_{\alpha\beta} q(\lambda n/E) | 0 \rangle, \quad (29)$$

in which $\epsilon_\perp^{\alpha\beta} = \epsilon^{\alpha\beta\gamma\lambda} n_\gamma \bar{n}_\lambda$. This form for $\hat{\phi}_\sigma(u)$ is convenient for our following calculations of the hard scattering kernels. The equations of motion for the q in the energetic limit is

$$i\not{D}(\lambda n/E) q(\lambda n/E) = 0 + O(1/E^2),$$

which is valid up-to $O(1/E^2)$. We finally arrive at Eq. (5).

By using $\hat{\phi}_\sigma(u)$, there is no the so-called projection ambiguity in the calculations of the hard scattering kernels, because there are no coordinate variables in the spin projector of $\hat{\phi}_\sigma(u)$. In the energetic meson limit, the ambiguity that the same $\phi_\sigma(u)$ is defined for different components with different magnitudes of the spin projector has been avoided. We conclude that $\hat{\phi}_p(u)$ and $\hat{\phi}_\sigma(u)$ are more appropriate than $\phi_p^\epsilon(u)$ and $\phi_\sigma^\epsilon(u)$ for uses in the calculation of the X_H term and similar twist-3 contributions.

The model for $\hat{\phi}_p(u)$ can be obtained by solving Eq. (5). Since the partons in the two parton Fock state of a light pseudoscalar meson can share the meson's momentum equally, we can assume that the parameterization models for $\hat{\phi}_p(u)$ and $\hat{\phi}_\sigma(u)$ can have the asymptotic form $6u(1-u)$, for simplicity. Because now the $\hat{\phi}_p(u)$ is no longer a constant, if the $\phi_p(u)$ in X_H can be replaced by $\hat{\phi}_p(u)$, then the end-point divergent problem can be resolved. However, this requires us to examine the contributions related to X_H term whether they are completely coming from the $\hat{\phi}_p(u)$ under the energetic meson limit. It is necessary to develop an appropriate calculation scheme for the twist-3 contributions.

III. THE COLLINEAR EXPANSION

As mentioned previously, the spin projector for $\phi_\sigma(u)$ contains a coordinate variable z . This leads to a projection ambiguity problem. For comparison, we will first review what this problem is. Next, we will introduce a collinear expansion method for calculations of the hard scattering kernels of the one loop corrections to the matrix element.

A. The spin projection ambiguity and infrared divergences

As mentioned in previous sections, the spin projector $[P_\alpha, z_\beta]$ for $\phi_\sigma(u)$ contains a coordinate variable z . There are two methods for performing the calculations of the contributions from the $\phi_\sigma(u)$. The first method proposed by Beneke and Neubert (BN)[20] is to separate the z in $[P_\alpha, z_\beta]$ into its collinear and transverse parts as

$$z_\beta \rightarrow (-i) \frac{\partial}{\partial k_\beta} = (-i) \left(\frac{n^\beta}{E} \frac{\partial}{\partial u} + \frac{\partial}{\partial k_{\perp\beta}} + \cdots \right), \quad (30)$$

where k is assumed to be the momentum carried by the quark. The $\partial/\partial u$ is defined to be applied on $\phi_\sigma(u)$, and the $\partial/\partial k_{\perp\beta}$ is interpreted to act on the associated hard scattering kernel for $\phi_\sigma(u)$. However, it is also legal to let the whole momentum derivative $\partial/\partial k_\beta$ applied on the hard scattering kernel. The latter method was proposed by Du, Yang, and Zhu (DYZ) [23, 24]. However, both methods cannot avoid the infrared divergence in the X_H term.

B. A preface to collinear expansion method

In this section, the CE calculation scheme proposed by Yeh in [57] will be used for calculations of $O(\alpha_s)$ contributions. The CE scheme is only applicable to calculate the decay amplitudes in the energetic meson limit. Under the CE scheme, the calculated twist-3 contributions are interpreted to be composed of collinear partons. A higher twist, which is composed of collinear partons, is usually called a dynamical power correction [56]. There exist other types of power corrections, such as the power corrections from soft gluon or renormalon contributions. We identify these as non-partonic power corrections. For these non-partonic power corrections, the CE may not be applicable. To include these non-partonic power corrections within QCDF requires further assumptions. For example, the soft gluon contributions are better determined by the QCD sum rules or the lattice QCD. In this work, we only investigate how the dynamical power corrections can be calculated by the CE method.

The idea of CE was first made by Polizer [58]. The systematical method was latter developed by Ellis, Furmanski and Petronzio (EFP) [54, 55]. Using CE, Ellis *et al* showed that, for DIS processes, the twist-4 power corrections (corrections suppressed by Q^{-2} with Q the relevant hard scale in the processes) can be factorized into its short distance and long distance

parts as the factorization of the leading twist contributions. However, a parton interpretation for the twist-4 power corrections is lost. To recover the parton model picture for the twist-4 power corrections, Qiu [56] introduced a Feynman-diagram approach to re-formulate the EFP's method. In this Feynman-diagram language, a parton model interpretation for the twist-4 power corrections becomes trivial.

C. Preliminary

The organization of this section is as following. We will first describe how CE can be applied for tree level diagrams. We then apply CE to calculate one loop corrections to the matrix elements under the two parton approximation. The factorization of the amplitudes of the one loop corrections will be shown up-to twist-3 order. In the following, we will use $1/E$ instead of using $1/m_b$ to represent the twist-3 order. To be specific, we shall consider the decay processes of a B meson into two pseudoscalar light mesons. The decay processes involve three restrictedly ordered energy scales: the W boson mass $\mu_W \sim M_W$ scale, the factorization scale $\mu_F \sim m_b$, and the characteristic energy scale of nonperturbative QCD Λ_{QCD} . With the help of operator product expansion (OPE), the relevant $|\Delta B| = 1$ effective Hamiltonian is given by

$$\begin{aligned}
H = & \frac{G_F}{\sqrt{2}} \left[\sum_{q=u,c} v_q (C_1(\mu) Q_1^q(\mu) + C_2(\mu) Q_2^q(\mu)) \right. \\
& + \sum_{k=3}^{10} C_k(\mu) Q_k(\mu) - v_t (C_{7\gamma}(\mu) Q_{7\gamma}(\mu) \\
& \left. + C_{8G}(\mu) Q_{8G}(\mu)) \right] + H.C. , \tag{31}
\end{aligned}$$

where $v_q = V_{qb}V_{qd}^*$ (for $b \rightarrow d$ transition) or $v_q = V_{qb}V_{qt}^*$ (for $b \rightarrow s$ transition) and $C_i(\mu)$ are the Wilson coefficients which have been evaluated to next-to-leading order approximation by means of perturbative QCD and renormalization group [70, 71, 72, 73]. The four quark operators Q_i are given by

$$\begin{aligned}
Q_1^u &= (\bar{u}_\alpha b_\alpha)_{V-A} (\bar{q}_\beta u_\beta)_{V-A} , Q_1^c = (\bar{c}_\alpha b_\alpha)_{V-A} (\bar{q}_\beta c_\beta)_{V-A} , \\
Q_2^u &= (\bar{u}_\alpha b_\beta)_{V-A} (\bar{q}_\beta u_\alpha)_{V-A} , Q_2^c = (\bar{c}_\alpha b_\beta)_{V-A} (\bar{q}_\beta c_\alpha)_{V-A} , \\
Q_3 &= (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V-A} , Q_4 = (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\alpha q'_\beta)_{V-A} , \\
Q_5 &= (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V+A} , Q_6 = (\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\alpha q'_\beta)_{V+A} ,
\end{aligned}$$

$$\begin{aligned}
Q_7 &= \frac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V+A} , \quad Q_8 = \frac{3}{2}(\bar{q}_\beta b_\alpha)_{V-A} (\sum_{q'} e_{q'} \bar{q}'_\alpha q'_\beta)_{V+A} , \\
Q_9 &= \frac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V-A} , \quad Q_{10} = \frac{3}{2}(\bar{q}_\beta b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\alpha q'_\beta)_{V-A} ,
\end{aligned} \tag{32}$$

and

$$\begin{aligned}
Q_{7\gamma} &= -\frac{e}{8\pi^2} m_b (\bar{q}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) b_\alpha) F_{\mu\nu} , \\
Q_{8G} &= -\frac{g}{8\pi^2} m_b (\bar{q}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) t_{\alpha\beta}^a b_\beta) G_{\mu\nu}^a ,
\end{aligned} \tag{33}$$

where Q_1^q and Q_2^q are the tree operators, $Q_3 - Q_6$ the QCD penguin operators, $Q_7 - Q_{10}$ the electroweak penguin operators, and $Q_{7\gamma}$ and Q_{8G} the magnetic and chromo-magnetic penguin operators.

The contributions between the scales of μ_F and μ_W are attributed to the Wilson coefficients, and the contributions between the scales of μ_F and Λ_{QCD} are then included into the matrix elements of the operators. By choosing an appropriate regularization method for the infrared singularities, the Wilson coefficients can be calculated to be independent of the external states and can be factorized from the matrix element $\langle M_1(p) M_2(q) | Q_i | \bar{B}(P_B) \rangle$. For an energetic M_2 meson in the $\bar{B} \rightarrow M_1 M_2$ processes, the color transparency leads to the naive factorization [69] for the matrix element such that the matrix element $\langle M_1(p) M_2(q) | Q_i | \bar{B}(P_B) \rangle$ can be written as a product of a transition form factor and a decay constant in the following way

$$\langle M_1(p) M_2(q) | Q_i | \bar{B}(P_B) \rangle \approx \langle M_1(p) | J_1^i | B(P_B) \rangle \langle M_2(q) | J_2^i | 0 \rangle + (M_1 \leftrightarrow M_2) . \tag{34}$$

The form factors are defined as

$$\begin{aligned}
&\langle M_1(p) | \bar{q}(0) \gamma^\mu b(0) | \bar{B} \rangle \\
&= F_+^{B \rightarrow M_1}(q^2) (P_B^\mu + p^\mu) + [F_0^{B \rightarrow M_1}(q^2) - F_+^{B \rightarrow M_1}(q^2)] \frac{m_B^2 - m_{M_1}^2}{q^2} q^\mu .
\end{aligned} \tag{35}$$

The form factors coincide as $q^2 = 0$, $F_+^{B \rightarrow M_1}(0) = F_0^{B \rightarrow M_1}(0)$. Due to the conservation of the currents $J_{1,2}^i$, the scale invariance of the decay amplitudes is broken under the naive factorization. The radiative corrections to the matrix elements are necessary to compensate the scale dependence of the matrix element [15, 17, 18].

The bottom meson momentum P_B is defined in a light-cone coordinate frame such that it can be written as $P_B^\mu = (p^\mu + q^\mu)$ with two light-like vectors $q^\mu = (q^+, q^-, q_\perp^i) = E\bar{n}^\mu$ and $p^\mu = (p^+, p^-, p_\perp^i) = En^\mu$ in the plus and minus directions, respectively. The M_1 meson is defined to receive the spectator quark of the bottom meson as its quark component. The M_2 meson is defined as the emitted meson from the weak interaction vertex. We associate the momentum p^μ for the M_1 meson and the momentum q^μ for the M_2 meson. The following metric and antisymmetric tensors are useful in our calculations

$$\begin{aligned} w_{\mu'}^\mu &= g_{\mu'}^\mu - \bar{n}^\mu n_{\mu'} , \quad \bar{w}_{\mu'}^\mu = g_{\mu'}^\mu - n^\mu \bar{n}_{\mu'} , \\ d_{\mu'}^\mu &= g_{\mu'}^\mu - \bar{n}^\mu n_{\mu'} - n^\mu \bar{n}_{\mu'} , \quad \bar{d}_{\mu'}^\mu = g_{\mu'}^\mu - n^\mu \bar{n}_{\mu'} - \bar{n}^\mu n_{\mu'} , \\ \epsilon_{\perp}^{\mu\nu} &= \epsilon^{\mu\nu\alpha\beta} n_\alpha \bar{n}_\beta , \quad \bar{\epsilon}_{\perp}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} \bar{n}_\alpha n_\beta . \end{aligned} \quad (36)$$

These tensors satisfy the following identities

$$q_\mu w_{\mu'}^\mu = 0 , \quad p_\mu \bar{w}_{\mu'}^\mu = 0 , \quad d_\mu^\mu = \bar{d}_\mu^\mu = 2 , \quad \epsilon_{\perp}^{\mu\nu} \epsilon_{\perp\mu\nu} = \bar{\epsilon}_{\perp}^{\mu\nu} \bar{\epsilon}_{\perp\mu\nu} = 2 . \quad (37)$$

For a collinear loop parton of the M_2 meson, it is convenient to parametrize its momentum l^μ into its components proportional to the meson momentum q^μ , the light-cone vector n^ν , and the transversal directions

$$l^\mu = E\bar{n}^\mu + \frac{l_\perp^2 + l_\perp^2}{2E} n^\mu + l_\perp^\mu . \quad (38)$$

For convenience, we further define the collinear component, \hat{l}^μ , the on shell component, l_L^μ , and the off shell component, l_S^μ , of the momentum l^μ as

$$\begin{aligned} \hat{l}^\mu &= n \cdot l \bar{n}^\mu , \\ l_L^\mu &= \hat{l}^\mu + \frac{l_\perp^2}{2n \cdot l} n^\mu + l_\perp^\mu , \\ l_S^\mu &= \frac{l_\perp^2}{2n \cdot l} n^\mu . \end{aligned} \quad (39)$$

For the M_1 meson, the momenta of the collinear loop partons flowing through the M_1 meson can be parametrized in a similar way. We parametrize the collinear momentum in terms of p^μ and \bar{n}^μ . Because the mass effects from the mesons' masses and the light quarks' masses are very small in the decay processes considered in this work, we shall neglect them completely in this paper. In this approximation, we let $m_{M_i} = 0$, $i = 1, 2$, and $q^2 = 0$. And the loop partons are assumed massless.

According to Eq. (38), a parton propagator can be separated into its long distance part and short distance part. If we write the loop parton propagator as

$$\begin{aligned} F(y, z) &= \int \frac{d^4 l}{(2\pi)^4} e^{i l \cdot (y-z)} [F_L(l) + F_S(l)] \\ &= F_L(y, z) + F_S(y, z) , \end{aligned} \quad (40)$$

where

$$F_L(l) = \frac{i \not{l}_L}{l^2 - i\epsilon} , \quad F_S(l) = \frac{i \not{n}}{2n \cdot l - i\epsilon} . \quad (41)$$

The $F_L(l)$ propagator corresponds to the long distance part of the propagator, since $F_L(y, z) \propto \theta(y - z)$. The $F_S(l)$ propagator represents the short distance part because $F_S(y, z) \propto \delta(y - z)$. We now describe one important property of the long distance propagator $F_L(l)$. As $F_L(l)$ contacts with a \not{n}^μ component of a vertex γ^μ in the parton amplitudes, the \not{n}^μ vertex will extract one short distance propagator $F_S(l)$ and one interaction vertex $i\gamma_\nu$ from the hadron amplitude in the following ways

$$\frac{i \not{l}_L}{l^2} \not{n} = \frac{i \not{l}_L}{l^2} (i\gamma_\nu) \frac{i \not{n}}{2n \cdot l} \not{n} (l - \hat{l})^\nu . \quad (42)$$

The momentum factor $(l - \hat{l})^\nu$ is then absorbed by the hadron amplitude due to the Ward identity [54, 55, 56]. The above identity is obtained by a simple manipulation [56]. We first insert an identity $1 = (\not{l}^2)/l^2$ into the left hand side of Eq. (42) and expand the \not{l} as $\not{l}_L + \not{l}_S$ as the following

$$\frac{i \not{l}_L}{l^2} \frac{\not{l}}{l^2} \not{n} = \frac{i \not{l}_L}{l^2} \frac{(\not{l}_L + \not{l}_S)(\not{l}_L + \not{l}_S)}{l^2} \not{n} \quad (43)$$

Since $(\not{l}_L)^2 = 0 = (\not{l}_S)^2$, we then obtain

$$\frac{i \not{l}_L}{l^2} \frac{\not{l}}{l^2} \not{n} = \frac{i \not{l}_L}{l^2} \frac{(\not{l}_L \not{l}_S + \not{l}_S \not{l}_L)}{l^2} \not{n} \quad (44)$$

where the first term $\not{l}_L \not{l}_S$ in the right hand side of the above equation leads to a vanishing result as it contacts with its left hand side $i \not{l}_L / l^2$ term. The only contribution can only come from the second term $\not{l}_S \not{l}_L$. We further expand the \not{l}_L in the following way

$$\not{l}_S \not{l}_L \not{n} = l^2 \frac{\not{n}}{2n \cdot l} (n \cdot l \not{n} + \frac{l_\perp^2 \not{n}}{2n \cdot l} + \not{l}_\perp) \not{n}$$

Due to $\not{n}^2 = \not{n}^2 = 0$, the remaining result becomes

$$l^2 \frac{\not{n}}{2n \cdot l} (\not{l}_\perp) \not{n} .$$

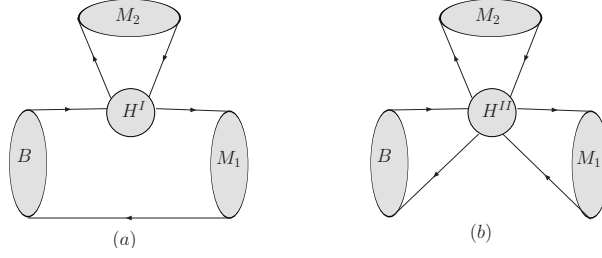


FIG. 1: (a) The type *I* diagram for $B \rightarrow M_1 M_2$ decays. (b) The type *II* diagram for $B \rightarrow M_1 M_2$ decays. The central blobs with symbols H^I, II represent the parton scattering functions. The initial state B meson is represented by the circle with a symbol B in the left hand side of each diagram. The final state M_1 meson is represented by the circle with a symbol M_1 in the right hand side of each diagram. The final state M_2 meson is represented by the circle with a symbol M_2 in the upper part of each diagram. The lines with a arrow are the fermion partons.

By substituting the above back into Eq. (43), Eq. (42) is derived by noting that

$$\not{l}_L(i\gamma_\alpha)(i\not{p})\not{q}(l - \hat{l})^\alpha = \not{l}_L\not{p}\not{l}_\perp\not{q}.$$

Using Eq. (42), one can systematically include the effects from the non-collinearity and off-shell-ness of collinear partons external to the hard scattering function of a Feynman diagram. This property of the long distance propagator plays an important role in our following analysis.

Let's first describe how the collinear expansion method can be applied for tree level amplitudes to recover the NF amplitudes. After the tree level analysis, we will investigate how CE method can be applied for the one loop amplitudes and show that the factorization theorem Eq. (1) up-to $O(1/E)$. We begin from the diagrams of Fig. 1 for the $\bar{B} \rightarrow M_1 M_2$ process. The parton interactions in the diagrams of Fig. 1 (a) and (b) are assumed by an effective four quark operator $Q_i = (\bar{q}_a \Gamma_i b)(\bar{q}_b \Gamma_i q_c)$. In order to discuss the collinear expansion, we propose to formally write the transition matrix element $\langle M_1 M_2 | Q_i | \bar{B} \rangle$ involved in the amplitude for the diagrams of Fig. 1 (a) and (b) as the following expression

$$\begin{aligned} \langle M_1 M_2 | Q_i | \bar{B} \rangle &= F_j^{B \rightarrow M_1}(q^2) \int \frac{d^4 l_{M_2}}{(2\pi)^4} \text{Tr}[H_{ij}^I(l_{M_2}) \phi^{M_2}(l_{M_2})] \\ &+ \int \frac{d^4 l_B}{(2\pi)^4} \int \frac{d^4 l_{M_1}}{(2\pi)^4} \int \frac{d^4 l_{M_2}}{(2\pi)^4} \text{Tr}[H_i^{II}(l_{M_2}) \phi_B(l_B) \phi^{M_1}(l_{M_1}) \phi^{M_2}(l_{M_2})] \\ &+ \dots, \end{aligned} \quad (45)$$

where $F_j^{B \rightarrow M_1}(q^2)$ represents the transition form factor denoted by the bottom part of the diagram Fig. 1 (a). The scattering functions $H^{I,II}$ contain the parton interactions. The H^I function represents four parton interactions with multiple radiative gluons. The H^{II} function represents six parton interactions with multiple radiative gluons. The dots denote higher order terms which could contain interactions between the three parton Fock state of M_2 and the other partons from the M_1 or B mesons. The contributions from the three parton Fock state of M_1 or M_2 are neglected completely in this paper. The dots' terms are not shown explicitly.

The diagrams in Fig. 1 (a) and (b) represent the processes that the initial state \bar{B} undertakes a transition into the final state M_1 by means of the $b \rightarrow q_1 q_2 \bar{q}_3$ decays accompanying multiple radiative interactions. The partonic radiative interactions associated with $b \rightarrow q_1 q_2 \bar{q}_3$ decays are collected into $H^{I,II}$. The probability for the transition of \bar{B} into $b\bar{q}_s$ pair is denoted by $\phi_B(l_B)$. The $q_2 \bar{q}_3$ pair produced from the interaction center, i.e. $H^{I,II}$, then combine to form the M_2 meson after a long distance travel away from the interaction center. The probability for the transition of the $q_2 \bar{q}_3$ pair into the M_2 is represented by $\phi^{M_2}(l_{M_2})$. In the diagram of Fig. 1 (a), the spectator quark \bar{q}_s of B combines with the q_1 produced from H^I to form the M_1 . The transition from \bar{B} into M_1 is represented by $F^{B \rightarrow M_1}$. In the diagram of Fig. 1 (b), the \bar{q}_s gets involved in H^{II} and then combines with q_1 to form M_1 . The probability for the transition of $\bar{q}_s q_1$ into M_1 is represented by $\phi^{M_1}(l_{M_1})$.

We first discuss the diagram in Fig. 1 (a), which is represented by the first term of Eq. (45). In the expression of the first term of Eq. (45), the scattering kernel $H_{ij}^I(l_{M_2})$ and the meson amplitude $\phi^{M_2}(l_{M_2})$ are correlated by the loop parton momentum l_{M_2} , the color indices, and the spin indices. The loop parton momentum l_{M_2} is defined to flow from the antiquark line to the quark line of M_2 . The expression for $\phi^{M_2}(l_{M_2})$ is defined as

$$\phi^{M_2}(l_{M_2}) = \int d^4 z e^{-i\bar{l}_{M_2} \cdot z} \langle M_2(q) | \bar{q}_2(0) q_3(z) | 0 \rangle, \quad (46)$$

where $\bar{l}_{M_2} = q - l_{M_2}$ and the color and spin indices are not shown explicitly. The Tr denote traces over the color and spin indices. To complete the factorization of the first term of Eq. (45) into the short distance and long distance parts, we need to disentangle the correlations in the integration over l_{M_2} , the color indices, and the spin indices.

In order to derive a factorization theorem similar to Eq. (1), we propose to employ the expansion scheme made in [57]. The $H_{ij}^I(l_{M_2})$ is expanded in α_s and $l_{M_2 \perp}$. Similarly, the

H_i^{II} is expanded in α_s and $l_{M_i\perp}$, $i = 1, 2$. Namely, we first expand $H_{ij}^I(l_{M_2})$ in α_s as

$$H_{ij}^I(l_{M_2}) = H_{ij}^{I(0)} + H_{ij}^{I(1)}(l_{M_2}) + \cdots, \quad (47)$$

$$H_i^{II}(l_{M_1}, l_{M_2}) = H_i^{II(1)}(l_{M_1}, l_{M_2}) + \cdots \quad (48)$$

where $H_{ij}^{I(0)}$ is of order $O(\alpha_s^0)$ and $H_{ij}^{I(1)}$ and $H_i^{II(1)}$ are of order $O(\alpha_s^1)$. The dots are of order $O(\alpha_s^n)$ with $n \geq 2$. Each term in Eq. (47) is then expanded in $l_{M_1\perp}$ or $l_{M_2\perp}$. The expansion of $H_{ij}^{I(1)}$ and $H_i^{II(1)}$ in $l_{M_1\perp}$ or $l_{M_2\perp}$ are left to latter discussions for one loop corrections.

D. Tree level expansion

The $H_{ij}^{I(0)}$ is independent of l_{M_2} and $H_{ij}^{I(0)} = \Gamma_i \delta_{ij} \delta^{ab}$. The color factor δ_{ab} is then absorbed by ϕ^{M_2} . By substituting $H_{ij}^{I(0)}$ back into the first term of Eq. (45), we arrive at

$$A_1 = F_j^{B \rightarrow M_1}(q^2) \int \frac{d^4 l_{M_2}}{(2\pi)^4} \text{Tr}[H_{ij}^{I(0)} \phi^{M_2}(l_{M_2})] . \quad (49)$$

We further use the integration transformation [54, 56]

$$1 = \int_0^1 dv \delta(v - l_{M_2} \cdot n / q \cdot n) = \int_0^1 dv \int_0^\infty \frac{d\lambda_2}{2\pi} e^{i\lambda_2(v - l_{M_2} \cdot n / E)} \quad (50)$$

to rewrite A_1 as

$$A_1 = F_j^{B \rightarrow M_1}(q^2) \int_0^1 dv \text{Tr}[H_{ij}^{I(0)} \phi^{M_2}(v)] . \quad (51)$$

The distribution amplitude $\phi^{M_2}(v)$ is given by

$$\phi^{M_2}(v) = \int_0^\infty \frac{d\lambda_2}{2\pi} e^{iv\lambda_2} \phi^{M_2}(\lambda_2) \quad (52)$$

where

$$\phi^{M_2}(\lambda_2) = \int d^4 z e^{-iq \cdot z} \int \frac{d^4 l_{M_2}}{(2\pi)^4} e^{il_{M_2} \cdot (z - \lambda_2 n / E)} \langle M_2(P_2) | \bar{q}_b(0) q_c(z) | 0 \rangle . \quad (53)$$

We further use the fact that the integrations over l_{M_2} and z result in a delta function

$$\int d^4 z \int \frac{d^4 l_{M_2}}{(2\pi)^4} e^{il_{M_2} \cdot (z - \lambda_2 n / E)} = \int d^4 z \delta^{(4)}(z - \lambda_2 n / E) . \quad (54)$$

By this, we write $\phi^{M_2}(v)$ as

$$\phi^{M_2}(v) = \int_0^\infty \frac{d\lambda_2}{2\pi} e^{-i\bar{v}\lambda_2} \langle M_2(q) | \bar{q}_2(0) q_3(\lambda_2 \frac{n}{E}) | 0 \rangle , \quad (55)$$

where $\bar{v} = 1 - v$. To factorize the remaining spin indices in the trace, we make use of the Fierz identity

$$\delta_{ij}\delta_{kl} = \frac{1}{4}((\gamma^\mu)_{ij}(\gamma_\mu)_{kl} + (\gamma^\mu\gamma_5)_{ij}(\gamma_5\gamma_\mu)_{kl} + (\gamma_5)_{ij}(\gamma_5)_{kl} + \frac{1}{2}(\sigma^{\mu\nu}\gamma_5)_{ij}(\sigma_{\mu\nu}\gamma_5)_{kl}) \quad (56)$$

to obtain

$$A_1 \simeq F_j^{B \rightarrow M_1}(q^2) \sum_k \int_0^1 dv \frac{1}{4} Tr[H_{ij}^{(0)} \Gamma'_k] Tr[\bar{\Gamma}'_k \phi^{M_2}(v)] , \quad (57)$$

where $\Gamma'_k \bar{\Gamma}'_k = (\gamma^\mu)(\gamma_\mu), (\gamma^\mu\gamma_5)(\gamma_5\gamma_\mu), (\gamma_5)(\gamma_5), \frac{1}{2}(\sigma^{\mu\nu}\gamma_5)(\sigma_{\mu\nu}\gamma_5)$ for $k = 1, \dots, 4$. To project onto the specific spin state of M_2 , we introduce the following definitions

$$\begin{aligned} Tr[\gamma^\mu \phi^{M_2}(v)] &= 0 , \\ Tr[\gamma^\mu \gamma_5 \phi^{M_2}(v)] &= -i f_{M_2} E \bar{n}^\mu \phi_P^{M_2}(v) , \\ Tr[\gamma_5 \phi_{M_2}(v)] &= -i f_{M_2} \mu_{M_2} \hat{\phi}_p^{M_2}(v) , \\ Tr[\sigma^{\mu\nu} \gamma_5 \phi^{M_2}(v)] &= -f_{M_2} \mu_{M_2} [\bar{n}^\mu, n^\nu] \hat{\phi}_\sigma^{M_2}(v) , \end{aligned} \quad (58)$$

where $\phi_P^{M_2}(v)$ is the twist-2 LCDA, $\hat{\phi}_p^{M_2}(v)$ and $\hat{\phi}_\sigma^{M_2}(v)$ are twist-3 LCDAs, and the LCDAs of twist order higher than three have been omitted. The final result depends on Dirac structure of the effective operator Q_i . The Dirac structure can be $\Gamma_i \otimes \Gamma_i = (\gamma_\mu(1 - \gamma_5)) \otimes (\gamma^\mu(1 \pm \gamma_5))$ for $(V - A)(V \pm A)$ operators, and $\Gamma_i \otimes \Gamma_i = -2((1 - \gamma_5)) \otimes ((1 + \gamma_5))$ for $-2(S - P)(S + P)$ operators. By substituting the Dirac matrices for the operators and using the definitions for the spin state of M_2 , the amplitude becomes

$$\begin{aligned} A_1^{(V-A)(V \pm A)} &= \pm i f_{M_2} F_+^{B \rightarrow M_1}(q^2) m_B^2 \int_0^1 dv \phi_P^{M_2}(v) , \\ A_1^{-2(S-P)(S+P)} &= i f_{M_2} F_+^{B \rightarrow M_1}(q^2) m_B^2 r_\chi^{M_2} \int_0^1 dv \hat{\phi}_p^{M_2}(v) , \end{aligned} \quad (59)$$

where $r_\chi^{M_2} = 2m_{M_2}^2 / (\bar{m}_b(\bar{m}_{q_2} + \bar{m}_{\bar{q}_3}))$ with $\bar{m}_b, \bar{m}_{q_2}, \bar{m}_{\bar{q}_3}$ being current quark masses. In the above example, we have present how the collinear expansion method can be applied for tree level diagrams. As a necessary condition, it can recover the result obtained by naive factorization (NF).

E. Factorizable one loop diagrams

The $O(\alpha_s)$ scattering function $H_{ij}^{I(1)}(l_{M_2})$ are classified into factorizable and non-factorizable parts

$$H_{ij}^{I(1)}(l_{M_2}) = H_{ij}^{I(1),F}(l_{M_2}) + H_{ij}^{I(1),NF}(l_{M_2}) . \quad (60)$$

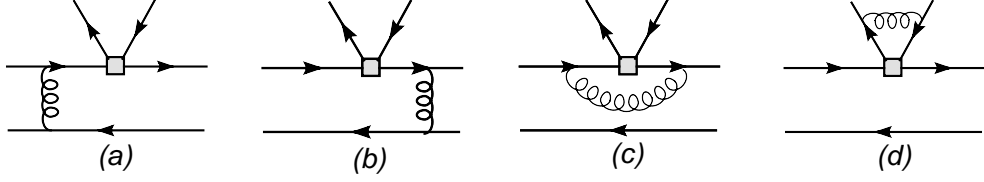


FIG. 2: The one loop factorizable diagrams. The external meson states are not shown.

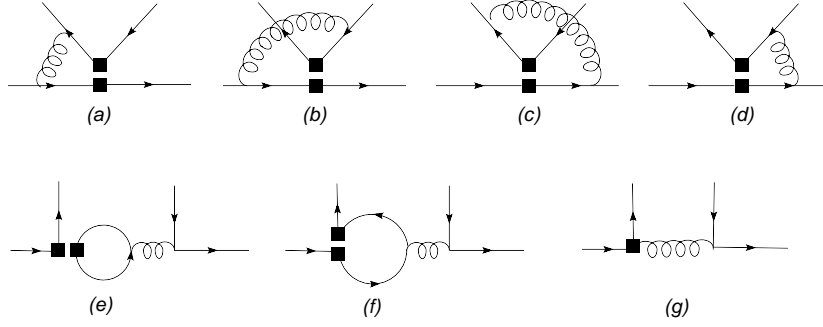


FIG. 3: (a)-(d) The vertex diagrams. (e)-(g) The penguin diagrams.

The $H_{ij}^{I(1),F}(l_{M_2})$ corresponds to the diagram in Fig. 2 (d). However, there are factorizable one loop diagrams as depicted in Fig. 2 (a)-(c), which are attributed to the $B \rightarrow M_1$ form factors $F_j^{B \rightarrow M_1}$. The sum of the first two diagrams Fig. 2 (a)-(b) for the form factor are dominated by soft gluons. Under QCD factorization approach, these leading soft contributions are defined to be absorbed by the physical form factors $F_j^{B \rightarrow M_1}$ [17]. The third factorizable diagram is to re-normalize the $V - A$ current associated with the $b \rightarrow q_1$ process. The fourth factorizable diagram is to re-normalize the $V - A$ current associated with the M_2 meson and the equations of motion. Since the $V - A$ current is conserved, the $V - A$ current and the equations of motion receive no renormalization. As a result, the factorizable diagrams lead to finite contributions under QCDF approach.

F. Non-factorizable one loop diagrams and collinear expansion

The $H_{ij}^{I(1),NF}(l_{M_2})$ corresponds to the non-factorizable one loop diagrams in Fig. 3 (a)-(g). According to the types of the diagrams, we write $H_{ij}^{I(1),NF}(l_{M_2})$ as

$$H_{ij}^{I(1),NF}(l_{M_2}) = V_{ij}(l_{M_2}) + P_{ij}(l_{M_2}) + P_{8g,ij}(l_{M_2}) , \quad (61)$$

where V_{ij} denotes the vertex corrections from the diagrams in Fig. 3 (a)-(d), P_{ij} denotes the penguin corrections from the diagrams in Fig. 3 (e)-(f), and $P_{8g,ij}$ denotes the magnetic dipole moment corrections from the diagrams in Fig. 3 (g). Each type of corrections will be analyzed in following each subsection, respectively.

We take the vertex correction $V_{ij}(l_{M_2})$ as an example to show the main feature of the application of the CE method to the one loop corrections. In the following, we omit the subscript M_2 in l_{M_2} to simplify the notation. We write the amplitude for the diagrams in Fig. 3 (a)-(d) as

$$A_V \sim F^{B \rightarrow M_1}(q^2) \int \frac{d^4 l}{(2\pi)^4} \text{Tr}[V(l)\phi^M(l)] . \quad (62)$$

In the above expression, the color and spin indices and the subscription of the form factor have been omitted for simplification. The irrelevant factors associated with the amplitude are also omitted for simplicity. To separate the collinear limiting $V(\hat{l})$ of $V(l)$ from the others, we make a Taylor expansion for $V(l)$ with respect to \hat{l} as

$$V(l) = V(\hat{l}) + \left. \frac{\partial V(l)}{\partial l^\alpha} \right|_{l=\hat{l}} (l - \hat{l})^\alpha + \dots , \quad (63)$$

where dots are higher derivative terms. The first derivative term is shown for our latter comparison with the BN and DYZ schemes and its related contribution will not be considered in this work. By using

$$w_{\alpha'}^\alpha = g_{\alpha'}^\alpha - \bar{n}^\alpha n_{\alpha'} ,$$

we can write

$$(l - \hat{l})^\alpha = w_{\alpha'}^\alpha l^{\alpha'} .$$

$l^{\alpha'}$ is then absorbed into $\phi^{M_2}(l)$ by Ward identity. With substitution of the first two terms of Eq. (63) back into Eq. (62), the result appears

$$A_V \sim \int \frac{d^4 l}{(2\pi)^4} \text{Tr}[V(v)\phi^{M_2}(l)] + \int \frac{d^4 l}{(2\pi)^4} \text{Tr}[V_\alpha(v, v)w_{\alpha'}^\alpha \phi_\partial^{M_2 \alpha'}(l, l)] + \dots , \quad (64)$$

where we have used the low energy theorems

$$V(v) \equiv V(\hat{l}) , \quad (65)$$

$$V_\alpha(v, v) \equiv \left. \frac{\partial V(l)}{\partial l^\alpha} \right|_{l=\hat{l}} , \quad (66)$$

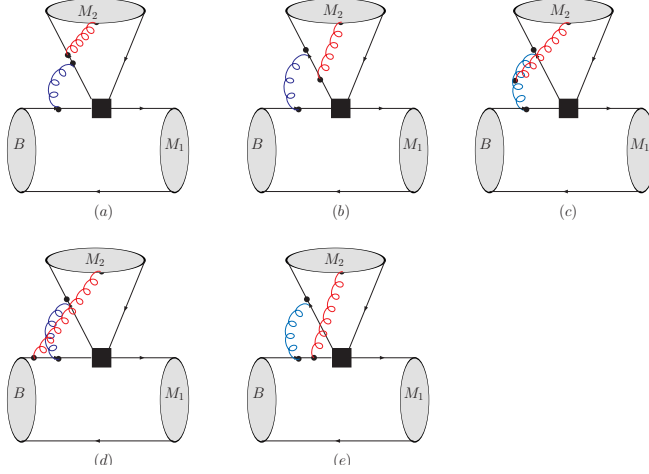


FIG. 4: The diagrams for $V_\alpha^{(1)}(\hat{l}, \hat{l}')$. For each diagram, there are similar diagrams with different gluon attachment.

and have defined

$$\phi_\partial^{M_2\alpha'}(l, l) = \int d^4z e^{-i\bar{l} \cdot z} \langle M_2(q) | \bar{q}(0) i\partial^{\alpha'}(z) q(z) | 0 \rangle. \quad (67)$$

The parameterization $\hat{l}^\mu = vE\bar{n}^\mu$ has been used. The v has the meaning of momentum fraction carried by the partons. The collinear momenta \hat{l} of the partons are defined to be parallel to the M_2 's momentum q in the \bar{n}^μ direction. The functions $V(v)$ and $V_\alpha(v, v)$ have been introduced to emphasize the fact that, under the collinear limiting, $V(\hat{l})$ and $V_\alpha(\hat{l}, \hat{l})$ and $\phi^{M_2}(l)$ are only correlated by v . Similar integral transformations to Eq. (50) can be used to rewrite Eq. (64) as

$$A_V \sim \int_0^1 dv \text{Tr}[V(v)\phi^{M_2}(v)] + \int_0^1 dv \text{Tr}[V_\alpha(v, v)w_{\alpha'}^\alpha \phi_\partial^{M_2\alpha'}(v, v)] + \dots, \quad (68)$$

where $\phi^{M_2}(v)$ and

$$\phi_\partial^{M_2\alpha'}(v, v) = \int_0^\infty \frac{d\lambda}{2\pi} e^{-i\lambda\bar{v}} \langle M_2(q) | \bar{q}_2(0) i\partial^{\alpha'}(\lambda \frac{n}{E}) q_3(\lambda \frac{n}{E}) | 0 \rangle \quad (69)$$

have been used.

For gauge invariance, we need to consider the diagrams as depicted in Fig. 4. We write the amplitude for the diagrams in Fig. 4 as

$$A'_V \sim F^{B \rightarrow M_1}(q^2) \int \frac{d^4l}{(2\pi)^4} \int \frac{d^4l'}{(2\pi)^4} \text{Tr}[V_\alpha(l, l')\phi_A^{M_2\alpha}(l, l')] , \quad (70)$$

where the function $V_\alpha(l, l')$ represents

$$V_\alpha(l, l') = V_\alpha^{(a)}(l, l') + V_\alpha^{(b)}(l, l') + V_\alpha^{(c)}(l, l') + V_\alpha^{(d)}(l, l') + V_\alpha^{(e)}(l, l') . \quad (71)$$

Each term in the right hand side of Eq. (71) corresponds to the diagrams as depicted in Fig. 4 (a)-(e), respectively. For each diagram in Fig. 4, there are other similar diagrams with different gluons' attachments. They are not shown explicitly. The amplitude $\phi_A^{M_2\alpha}(l, l')$ is defined as

$$\phi_A^{M_2\alpha}(l, l') = \int d^4y \int d^4z e^{-i(\vec{l}' - \vec{l}) \cdot y} e^{-i\vec{l} \cdot z} \langle M_2(q) | \bar{q}(0)(-) g A^{\alpha,a}(y) T^a q(z) | 0 \rangle . \quad (72)$$

In the following, we only consider the contributions related to $V_\alpha^{(b)}(l, l')$, which results in contributions related to $\langle M_2 | \bar{q}(0) i D^\mu(z') q(z) | 0 \rangle$. The other contributions, $V_\alpha^{(i)}$, $i \neq b$, are related to the $\langle M_2 | \bar{q}(0) G^{\mu\nu}(z') q(z) | 0 \rangle$. The twist-3 contributions related to $\langle M_2 | \bar{q}(0) G^{\mu\nu}(z') q(z) | 0 \rangle$ are left to our another preparing paper.

The collinear limiting part of $V_\alpha^{(b)}(l, l')$ is derived by using CE

$$V_\alpha^{(b)}(l, l') = V_\alpha^{(b)}(\hat{l}, \hat{l}') + \sum_{k=l, l'} \left. \frac{\partial V_\alpha^{(b)}(l, l')}{\partial k^\beta} \right|_{k=\hat{k}} (k - \hat{k})^\beta + \dots , \quad (73)$$

By substituting Eq. (73) into Eq. (70) and neglecting the other terms except of $V_\alpha^{(b)}(\hat{l}, \hat{l}')$, we arrive at

$$A'_V \sim F^{B \rightarrow M_1}(q^2) \int \frac{d^4l}{(2\pi)^4} \int \frac{d^4l'}{(2\pi)^4} \text{Tr}[V_\alpha^{(b)}(\hat{l}, \hat{l}') \phi_A^{M_2\alpha}(l, l')] . \quad (74)$$

Further more, it is convenient to rewrite $A^\alpha(z')$ in $\phi_A^{M_2\alpha}(l, l')$ as

$$A^\alpha(z') = \bar{n}^\alpha n \cdot A(z') + w_\alpha^\alpha A^{\alpha'}(z') . \quad (75)$$

By substituting the above expansion for $A^\alpha(z')$ into Eq. (74), we arrive at

$$\begin{aligned} A'_V \sim F^{B \rightarrow M_1}(q^2) & \int \frac{d^4l}{(2\pi)^4} \int \frac{d^4l'}{(2\pi)^4} \\ & \times \{ \text{Tr}[V_\alpha^{(b)}(\hat{l}, \hat{l}') \bar{n}^\alpha n_{\alpha'} \phi_A^{M_2\alpha'}(l, l')] \\ & + \text{Tr}[V_\alpha^{(b)}(\hat{l}, \hat{l}') w_\alpha^\alpha \phi_A^{M_2\alpha'}(l, l')] \} . \end{aligned} \quad (76)$$

For covariant gauge, the first term of Eq. (76) can be transferred in the gauge phase factor and the second term of Eq. (76) can be combined with the second term of Eq. (64) (See below further explanations.). For physical gauge, such as $n \cdot A = 0$, it is automatically

vanishing. In this work, we choose to use the covariant gauge in our following analysis. Under the covariant gauge, there are other contributions from diagrams with more partonic gluons of M_2 . The collinear limitings of these contributions are equally important and should be considered. However, they can be shown to contribute to the gauge phase factor. In the following, we exhibit this fact by only considering one partonic gluon case, the first term of Eq. (76). We now explain how the first term of Eq. (76) can be transferred into a gauge phase factor. By using the identity

$$V_\alpha^{(b)}(\hat{l}, \hat{l}') \bar{n}^\alpha = V_\alpha^{(b)}(\hat{l}, \hat{l}') \frac{k^\alpha}{n \cdot k} \Big|_{k=(\hat{l}'-\hat{l})} \quad (77)$$

and the Feynman identity

$$k^\alpha V_\alpha^{(b)}(\hat{l}, \hat{l}') = V(\hat{l}) - V(\hat{l}') , \quad (78)$$

we can rewrite the first term of Eq. (76) in the following form

$$\int \frac{d^4 l}{(2\pi)^4} \int \frac{d^4 l'}{(2\pi)^4} \left(\text{Tr}[V(\hat{l}) \phi_{n \cdot A}^{M_2}(l, l')] - \text{Tr}[V(\hat{l}') \phi_{n \cdot A}^{M_2}(l, l')] \right) , \quad (79)$$

where

$$\phi_{n \cdot A}^{M_2 \alpha}(l, l') = \int d^4 y \int d^4 z e^{ik \cdot (y - n\eta)} e^{-i\bar{l} \cdot z} \langle M_2(q) | \bar{q}(0) (-ig) \int_0^\infty d\eta n \cdot A^a(y) T^a q(z) | 0 \rangle . \quad (80)$$

in which the identity

$$\frac{i}{n \cdot k - i\epsilon} = \int_0^\infty d\eta e^{-i\eta n \cdot k} \quad (81)$$

has been used. Completing the momentum and coordinate integrals, we obtain

$$\int dv' \text{Tr}[V^{(1)}(v) \phi_{n \cdot A}^{M_2}(v)] - \int du \text{Tr}[V^{(1)}(v') \phi_{n \cdot A}^{M_2}(v')] , \quad (82)$$

where

$$\phi_{n \cdot A}^{M_2}(v) = \int_0^\infty \frac{d\lambda}{2\pi} e^{-i\bar{v}\lambda} \langle M_2(q) | \bar{q}(0) (-ig) \int_0^\infty d\eta n \cdot A^a(\eta n/E) T^a q(\lambda n/E) | 0 \rangle . \quad (83)$$

Since $V_\alpha(\hat{l}, \hat{l})$ and $V_\alpha^{(b)}(\hat{l}, \hat{l}')$ have similar structures, this enable us to add up A_V and A'_V to obtain

$$\begin{aligned} A_V + A'_V &\sim F^{B \rightarrow M_1}(q^2) \\ &\times \left\{ \int \frac{d^4 l}{(2\pi)^4} \text{Tr}[V(\hat{l}) \phi^{M_2}(l)] \right. \\ &\quad \left. + \int \frac{d^4 l}{(2\pi)^4} \int \frac{d^4 l'}{(2\pi)^4} \text{Tr}[V_\alpha^{(b)}(\hat{l}, \hat{l}') w_{\alpha'}^\alpha \phi_D^{M_2 \alpha'}(l, l')] \right\} \end{aligned} \quad (84)$$

where

$$\phi_D^{M_2\alpha}(l, l') = \int d^4y \int d^4z e^{-i(\vec{l}' - \vec{l}) \cdot y} e^{-i\vec{l} \cdot z} \langle M_2(q) | \bar{q}(0) iD^\alpha(y) q(z) | 0 \rangle . \quad (85)$$

with $iD^\alpha = i\partial^\alpha - gA^\alpha$. By using the integral transformations for l and l' , we arrive at

$$\begin{aligned} A + A' \sim & F^{B \rightarrow M_1}(q^2) \times \left(\int_0^1 dv \text{Tr}[V(v) \phi^{M_2}(v)] \right. \\ & \left. + \int_0^1 dv \int_0^1 dv' \text{Tr}[V_\alpha^{(b)}(v, v') w_\alpha^\alpha \phi_D^{M_2\alpha'}(v, v')] \right) , \end{aligned} \quad (86)$$

where

$$\begin{aligned} V(v) &\equiv V(\hat{l}) , \\ V_\alpha^{(b)}(v, v') &\equiv V_\alpha^{(b)}(\hat{l}, \hat{l}') , \\ \phi^{M_2}(v) &\equiv \int_0^\infty \frac{d\lambda}{2\pi} e^{-i\bar{v}\lambda} \langle M_2(q) | \bar{q}(0) q(\lambda n/E) | 0 \rangle , \\ \phi_D^{M_2\alpha'}(v, v') &\equiv \int_0^\infty \frac{d\eta}{2\pi} \int_0^\infty \frac{d\lambda}{2\pi} e^{-i(\bar{v}' - \bar{v})\eta} e^{-i\bar{v}\lambda} \langle M_2(q) | \bar{q}(0) iD^{\alpha'}(\eta n/E) q(\lambda n/E) | 0 \rangle . \end{aligned} \quad (87)$$

The contributions associated with the $V_\alpha^{(b)}(v, v')$ are at least of $O(1/E^2)$ and will be neglected. In the above, we have written the term related to $V_\alpha^{(b)}(v, v')$. It is given here for comparison of the CE scheme with the BN and DYZ schemes. We have introduced how the CE is applied to derive the contributions related to the collinear limiting part $V(v)$ and how the CE can separate different contributions related to different number of collinear partons of M_2 . Because we are only concerning the twist-3 contributions, our remaining task is to show that $V(v)$ is infrared finite up-to $O(1/E)$. This is given below.

G. Collinear expansion for vertex corrections

The relevant term in the amplitude for the four diagrams depicted in Fig. 3 (a)-(d) is written as

$$A_V^{Q_i} \sim F_j^{B \rightarrow M_1}(q^2) \int \frac{d^4 l_2}{(2\pi)^4} \text{Tr}[V_{ij}(l) \phi^{M_2}(l)] \quad (88)$$

where

$$\begin{aligned} \text{Tr}[V_{ij}(l) \phi^{M_2}(l)] &= 2i \frac{\pi\alpha_s}{N_c} \int \frac{d^4 k}{(2\pi)^4} \text{Tr}[T^a T^b \bar{\Gamma}_j(\Gamma_i \frac{(2P_{b,\alpha} - \gamma_\alpha \not{k})}{2P_b \cdot k + k^2} - \frac{(2p_\alpha + \not{k}\gamma_\alpha)}{2p \cdot k - k^2} \Gamma_i)] \\ &\times \text{Tr}[T^a T^b (\gamma^\alpha \frac{(\not{l} + \not{k})}{(l+k)^2} \bar{\Gamma}_i - \bar{\Gamma}_i \frac{(\bar{l} + \not{k})}{(\bar{l} + k)^2} \gamma^\alpha) \phi^{M_2}(l)] \frac{1}{k^2} , \end{aligned} \quad (89)$$

where we have employed equations of motion for the b and q_1 quarks for the terms inside the first trace bracket. The virtual gluon's momentum is represented by k . We first perform the following expansion for $V_{ij}(l)$ to derive its collinear limiting part $V(\hat{l})$

$$V(l) = V(\hat{l}) + \left. \frac{\partial V}{\partial l^\alpha} \right|_{l=\hat{l}} (l - \hat{l})^\alpha + \dots \quad (90)$$

The virtual gluons could be hard $k_H^\mu \sim (E, E, E)$, soft $k_S^\mu \sim (\lambda, \lambda, \lambda)$, or collinear to q^μ as $k_C^\mu \sim (E, \lambda^2/E, \lambda)$ or collinear to p^μ as $k_{C'}^\mu \sim (\lambda^2/E, E, \lambda)$. Therefore, we divide the k integral into three regions corresponding to the soft k_S , the collinear k_C or $k_{C'}$, and the hard k_H .

To analyze the infrared structure of $V^{(1)}(\hat{l})$, we define the soft and collinear limiting parts of $V_{ij}^{(1)}(\hat{l})$, in which k are set as soft k_S or collinear k_C or $k_{C'}$, as $V_{ij,S}^{(1)}(\hat{l})$ and $V_{ij,C}^{(1)}(\hat{l})$ and $V_{ij,C'}^{(1)}(\hat{l})$, respectively.

1. Soft finiteness

We first write $\text{Tr}[V_{ij,S}^{(1)}(\hat{l})\phi^{M_2}(l)]$ in its explicit form as

$$\begin{aligned} \text{Tr}[V_{ij,S}^{(1)}(\hat{l})\phi^{M_2}(l)] &= i \frac{C_F \pi \alpha_s}{N_c} \int \frac{d^4 k_S}{(2\pi)^4} \text{Tr}[\bar{\Gamma}_j \Gamma_i (\frac{2P_{b,\alpha}}{2P_b \cdot k_S} - \frac{2p_\alpha}{2p \cdot k_S})] \\ &\times \text{Tr}[(\gamma^\alpha \frac{\hat{l} + \not{k}_S}{(\hat{l} + k_S)^2} \bar{\Gamma}_i - \bar{\Gamma}_i \frac{\hat{l} + \not{k}_S}{(\hat{l} + k_S)^2} \gamma^\alpha) \phi^{M_2}(l)] \frac{1}{k_S^2}. \end{aligned} \quad (91)$$

In order to find the $O(1/E)$ contributions in the above expression, we have written the full part (i.e. $\hat{l} + k_s$ or $\hat{l} + k_s$) of the internal parton propagators in the upper part of the diagrams in Fig. 3 (a)-(d). Take an example for explanation, we consider the propagator for the left internal parton propagator in the upper part of the first diagram in Fig. 3 (a). Because $k_S^2 \ll \hat{l} \cdot k_S$, the propagator appears as

$$\frac{\hat{l} + \not{k}_S}{(\hat{l} + k_S)^2} = \frac{\hat{l}}{2\hat{l} \cdot k_S} + \frac{\not{n}}{2n \cdot \hat{l}} + \frac{\not{t}_\perp}{2n \cdot \hat{l}} \quad (92)$$

where t_\perp^μ is a unit vector in the transverse directions, $t_\perp^2 = 1$. Since the denominator of the first term in Eq. (92) is of $O(E\Lambda)$, and the denominators of the last two terms are of $O(E)$, we obtain the first term is of $O(1/\Lambda)$ and the last two terms are of $O(1/E)$. Since $E \gg \Lambda$, the leading contribution comes from the first term and the last two terms are power suppressed as $O(\Lambda/E)$ in comparison with the leading first term. Since the last two terms

are independent of k_S , they can be decomposed from the loop momentum integrations over k_S .

After we have separated the leading and subleading terms, where the latter are power and soft suppressed as $O(\Lambda/E)$ than the former, we now show that the leading part of V_S gives a vanishing result. By observing Eq. (92), we can see that the relevant terms of V_S are

$$\text{Tr}[\bar{\Gamma}_j \Gamma_i (\frac{2P_{b,\alpha}}{2P_b \cdot k_S} - \frac{2p_\alpha}{2p \cdot k_S})] \text{Tr}[(\gamma^\alpha \frac{\hat{\not{y}}}{2\hat{l} \cdot k_S} \bar{\Gamma}_i - \bar{\Gamma}_i \frac{\hat{\not{y}}}{2\hat{l} \cdot k_S} \gamma^\alpha) \phi^{M_2}(l)] \frac{1}{k_S^2}. \quad (93)$$

The γ^α in the second trace term can be $\gamma^\alpha = \not{n} \bar{n}^\alpha$, $\not{n} n^\alpha$, or, $d_{\alpha'}^{\alpha'} \gamma^{\alpha'}$. If $\gamma^\alpha = \bar{n} n^\alpha$ or $\not{n} \bar{n}^\alpha$, the second trace vanishes as

$$\begin{aligned} & \text{Tr}[(\not{n} n^\alpha \frac{\hat{\not{y}}}{2\hat{l} \cdot k_S} \bar{\Gamma}_i - \bar{\Gamma}_i \frac{\hat{\not{y}}}{2\hat{l} \cdot k_S} \not{n} n^\alpha) \phi^{M_2}(l)] \\ &= \text{Tr}[(\not{n} \bar{n}^\alpha \frac{\hat{\not{y}}}{2\hat{l} \cdot k_S} \bar{\Gamma}_i - \bar{\Gamma}_i \frac{\hat{\not{y}}}{2\hat{l} \cdot k_S} \not{n} \bar{n}^\alpha) \phi^{M_2}(l)] \\ &= 0, \end{aligned} \quad (94)$$

where the first line vanishes due to $\not{n} \hat{l} \propto \bar{n}^2 = 0$, and the second line vanishes since $\not{n} \hat{\not{y}} = vE$, $\not{n} \hat{\not{y}} = \bar{v}E$ and

$$\begin{aligned} & (\frac{\not{n} \hat{\not{y}}}{2n \cdot \hat{l}} \bar{\Gamma}_i - \bar{\Gamma}_i \frac{\hat{\not{y}} \not{n}}{2n \cdot \hat{l}}) \bar{n}^\alpha \\ &= \frac{\bar{n}^\alpha}{2} (\bar{\Gamma}_i - \Gamma_i) = 0. \end{aligned} \quad (95)$$

If $\gamma^\alpha = d_{\alpha'}^{\alpha'} \gamma^{\alpha'}$, the contraction of $d_{\alpha'}^{\alpha'}$ with the first trace gives a vanishing result as

$$d_{\alpha'}^{\alpha'} \left(\frac{2P_{b,\alpha}}{2P_b \cdot k_S} - \frac{2p_\alpha}{2p \cdot k_S} \right) = 0.$$

The $O(\Lambda/E)$ contributions from Eq. (92) are related to the spin state of M_2 . Because up-to twist-3 order, $q_2 \bar{q}_3$ pair for the M_2 can be proportional to $\not{n} \gamma_5$, γ_5 , or $\epsilon_{\perp}^{\alpha\beta} \sigma_{\alpha\beta}$, the substitution of these spin terms into the trace over those $O(\Lambda/E)$ terms in the internal propagators, such as those last two terms in the right hand side of Eq. (92), results in vanishing results by an explicit manipulation. This concludes that the $O(1/E)$ soft contributions vanish and the uncertainties are of $O(1/E^2)$. In summary, we have shown that the $V_{ij,S}^{(1)}(\hat{l})$ vanishes up-to $O(1/E)$ explicitly. The uncertainties are estimated to be of $O(1/E^2)$ which has beyond our precision used in this paper.

2. Collinear finiteness

We now consider the collinear part $V_{ij,C}^{(1)}(\hat{l})$. It is convenient to combine $\hat{l}^\mu + k_C^\mu = l'^\mu$

$$\begin{aligned} \text{Tr}[V_{ij,C}^{(1)}(\hat{l})\phi^{M_2}(l_2)] &= i\frac{C_F\pi\alpha_s}{N_c} \int \frac{d^4k_C}{(2\pi)^4} \text{Tr}[\bar{\Gamma}_j(\frac{\Gamma_i(2P_{b,\alpha} - \gamma_\alpha \not{k}_C)}{2P_b \cdot k_C} - \frac{(2p_\alpha + \not{k}_C\gamma_\alpha)}{2p \cdot k_C}\Gamma_i)] \\ &\times \text{Tr}[(\gamma^\alpha \frac{\not{l}'}{(l')^2}\bar{\Gamma}_i - \bar{\Gamma}_i \frac{\not{l}'}{(l')^2}\gamma^\alpha)\phi^{M_2}(l_2)] \frac{1}{k_C^2}. \end{aligned} \quad (96)$$

We separate the internal parton propagators into the long distance and short distance parts, and analyze their contributions, respectively. Let's first consider the term

$$\text{Tr}[(\gamma^\alpha(\frac{\not{l}_L}{l'^2} + \frac{\not{l}}{2n \cdot \hat{l}})\bar{\Gamma}_i - \bar{\Gamma}_i(\frac{\not{l}_L}{(l')^2} + \frac{\not{l}}{2n \cdot \hat{l}})\gamma^\alpha)\phi^{M_2}(l)], \quad (97)$$

where we have explicitly written the long distance term

$$\text{Tr}[(\gamma^\alpha \frac{\not{l}_L}{l'^2}\bar{\Gamma}_i - \bar{\Gamma}_i \frac{\not{l}_L}{(l')^2}\gamma^\alpha)\phi^{M_2}(l)], \quad (98)$$

and the short distance term

$$\text{Tr}[(\gamma^\alpha \frac{\not{l}}{2n \cdot \hat{l}'}\bar{\Gamma}_i - \bar{\Gamma}_i \frac{\not{l}}{2n \cdot \hat{l}}\gamma^\alpha)\phi^{M_2}(l)]. \quad (99)$$

Because the short distance part is suppressed than the leading part of the long distance term by an $O(\Lambda^2/E^2)$ factor, we may safely ignore the contributions associated with the short distance term.

In Eq. (98), the γ^α can be $\gamma^\alpha = \not{n}\bar{n}^\alpha$, $\not{n}n^\alpha$, or, $d_{\alpha'}^\alpha\gamma^{\alpha'}$. The final result also depends on the spin state of the M_2 . Because up-to twist-3 order, $q_2\bar{q}_3$ pair for the M_2 can be proportional to $\not{n}\gamma_5$, γ_5 , or $\epsilon_\perp^{\alpha\beta}\sigma_{\alpha\beta}$, the substitution of these spin terms into the second trace term Eq. (96) leads to the following nine results. We explain them term by term:

- $\gamma^\alpha = \not{n}n^\alpha$ and $[q_2\bar{q}_3] \propto \not{n}\gamma_5$: the trace term in Eq. (98) vanishes due to $\bar{n}^2 = 0$.
- $\gamma^\alpha = \not{n}\bar{n}^\alpha$ and $[q_2\bar{q}_3] \propto \not{n}\gamma_5$: the trace term in Eq. (98) is proportional to \bar{n}^α . The contraction of \bar{n}^α with the first trace term in Eq. (96) gives

$$\bar{n}^\alpha \text{Tr}[\bar{\Gamma}_j(\Gamma_i \frac{(2P_{b,\alpha} - \gamma_\alpha \not{k}_C)}{2P_b \cdot k_C} - \frac{(2p_\alpha + \not{k}_C\gamma_\alpha)}{2p \cdot k_C}\Gamma_i)] \simeq 0 + O(\frac{\Lambda^2}{E^3}). \quad (100)$$

This is because $\bar{n} \cdot P_b = \bar{n} \cdot p$ and $P_b \cdot k_C \simeq p \cdot k_C + O(\Lambda^2)$. The errors are from the \not{n} component of \not{k}_C .

- $\gamma^\alpha = d_{\alpha'}^{\alpha'} \gamma^{\alpha'}$ and $[q_2 \bar{q}_3] \propto \not{p} \gamma_5$: the trace term in Eq. (98)

$$d_{\alpha'}^\alpha \text{Tr}[(\gamma^{\alpha'} \frac{\not{p}_L}{\not{p}^2} \bar{\Gamma}_i - \bar{\Gamma}_i \frac{\not{p}_L}{(\not{p}')^2} \gamma^{\alpha'} \not{p} \gamma_5)] \propto d_{\alpha'}^\alpha \quad (101)$$

is proportional to $d_{\alpha'}^\alpha$. The leading contributions of the trace term in Eq. (98) are of $O(E/\Lambda^2)$. The contraction of $d_{\alpha'}^\alpha$ with the first trace term in Eq. (96) results in

$$\begin{aligned} & d_{\alpha'}^\alpha \text{Tr}[\bar{\Gamma}_j (\Gamma_i \frac{(2P_{b,\alpha} - \gamma_\alpha \not{k}_C)}{2P_b \cdot k_C} - \frac{(2p_\alpha + \not{k}_C \gamma_\alpha)}{2p \cdot k_C} \Gamma_i)] \\ & \propto \frac{d_{\alpha'}^\alpha k_{C,\alpha}}{p \cdot k_C}, \end{aligned} \quad (102)$$

which is of order $O(\Lambda/E^2)$. Due to the loop integration over k_C , the single $k_{C,\perp}$ factor selects another $k_{C,\perp}$ factor from the second trace term in Eq. (96). The product of these two trace terms are of $O(\Lambda^2/E^3)$.

- $\gamma^\alpha = \not{p} n^\alpha$ and $[q_2 \bar{q}_3] \propto \gamma_5$: the trace term in Eq. (98) is then proportional to

$$(\frac{1}{n \cdot \hat{p}'} - \frac{1}{n \cdot \bar{p}'}) n^\alpha. \quad (103)$$

The contraction of n^α with the first trace term in Eq. (96) gives

$$\begin{aligned} & n^\alpha \text{Tr}[\bar{\Gamma}_j (\Gamma_i \frac{(2P_{b,\alpha} - \gamma_\alpha \not{k}_C)}{2P_b \cdot k_C} - \frac{(2p_\alpha + \not{k}_C \gamma_\alpha)}{2p \cdot k_C} \Gamma_i)] \\ & \propto \frac{1}{p \cdot k_C} (P_b \cdot n - k_c \cdot n) = \frac{\bar{\alpha}}{\alpha E}, \end{aligned} \quad (104)$$

where we have used the fact that $\Gamma_i \otimes \bar{\Gamma}_i$ should be $-2(S - P) \otimes (S + P)$ and the parameterization $k_C^\mu = \alpha E \bar{n}^\mu + \dots$. The combination of these two trace terms are of $O(1/E^2)$.

- $\gamma^\alpha = \not{p} \bar{n}^\alpha$ and $[q_2 \bar{q}_3] \propto \gamma_5$: the trace term in Eq. (98) vanishes similar to Eq. (94).
- $\gamma^\alpha = d_{\alpha'}^{\alpha'} \gamma^{\alpha'}$ and $[q_2 \bar{q}_3] \propto \gamma_5$: the trace term in Eq. (98) is proportional to $k_{C,\perp}^\alpha$ and is of order $O(\Lambda/E)$. The contraction of $k_{C,\perp}^\alpha$ with the first trace term in Eq. (96) gives contributions of order $O(\Lambda/E^2)$. The product of these two trace terms is of order $O(\Lambda^2/E^3)$. We neglect it.
- $\gamma^\alpha = \not{p} n^\alpha$ and $[q_2 \bar{q}_3] \propto \epsilon_\perp^{\mu\nu} \sigma_{\mu\nu}$: only $\Gamma_i \otimes \bar{\Gamma}_i = -2(S - P) \otimes (S + P)$ can contribute. The second trace term in Eq. (98) gives a result proportional to

$$(\frac{1}{n \cdot \hat{p}'} + \frac{1}{n \cdot \bar{p}'}) \bar{n}^\alpha \quad (105)$$

which is of order $O(\Lambda/E)$. The contraction of \bar{n}^α with the first trace term in Eq. (96) gives a contribution of $O(1/E)$. The combination of these two trace terms is of order $O(\Lambda/E^2)$.

- $\gamma^\alpha = \not{n}\bar{n}^\alpha$ and $[q_2\bar{q}_3] \propto \epsilon_\perp^{\mu\nu}\sigma_{\mu\nu}$: only $\Gamma_i \otimes \bar{\Gamma}_i = -2(S - P) \otimes (S + P)$ can contribute. The trace term in Eq. (98) is proportional to \bar{n}^α . The contraction of \bar{n}^α with the first trace term in Eq. (96) gives a vanishing result with errors of $O(\Lambda^2/E^3)$.

In summary, the $\text{Tr}[V_C(\hat{l}_2)\phi^{M_2}(l)]$ vanishes up-to $O(1/E)$. Similarly, we can show that the term $\text{Tr}[V_{ij,C'}(\hat{l})\phi^{M_2}(l)]$ vanishes up-to $O(1/E)$. Therefore, up-to $O(1/E)$, we can neglect collinear divergences. Note that the above estimated errors should be multiplied by a factor $\Lambda^2 E$ for the $[q_2\bar{q}_3]_{M_2}$ spin state being proportional to $\not{n}\gamma_5$, and $\Lambda^2\mu_{M_2}$ for the γ_5 and $\epsilon_\perp \cdot \sigma$ spin states, respectively.

3. Infrared finite one loop vertex corrections

Based on the above analysis for the soft and collinear limitings of $V(\hat{l})$, we now describe how to use the subtraction method to show the infrared finiteness of $\text{Tr}[V(\hat{l})\phi^{M_2}(l)]$. The proof is given by the following series of identities

$$\begin{aligned}
A_V^{Q_i} &= F_j^{B \rightarrow M_1}(q^2) \int \frac{d^4 l}{(2\pi)^4} \text{Tr}[V_{ij}(l)\phi^{M_2}(l)] \\
&= F_j^{B \rightarrow M_1}(q^2) \int \frac{d^4 l}{(2\pi)^4} (\text{Tr}[(V_{ij}(\hat{l})\phi^{M_2}(l)] + \dots) \\
&= F_j^{B \rightarrow M_1}(q^2) \int \frac{d^4 l}{(2\pi)^4} [(\text{Tr}[V_{ij}(\hat{l})\phi^{M_2}(l)] - \text{Tr}[V_{ij;S}(\hat{l})\phi^{M_2}(l)] - \text{Tr}[V_{ij;C}(\hat{l})\phi^{M_2}(l)] \\
&\quad - \text{Tr}[V_{ij;C'}(\hat{l})\phi^{M_2}(l)]) + \text{Tr}[V_{ij;S}(\hat{l})\phi^{M_2}(l)] + \text{Tr}[V_{ij;C}(\hat{l})\phi^{M_2}(l)] \\
&\quad + \text{Tr}[V_{ij;C'}(\hat{l})\phi^{M_2}(l)] + \dots] \\
&= F_j^{B \rightarrow M_1}(q^2) \int_0^1 dv (\text{Tr}[V_{ij;H}(v)\phi^{M_2}(v)] + \dots) ,
\end{aligned} \tag{106}$$

where the $V_{ij;H}(v)$ is defined as

$$V_{ij;H}(v) = V_{ij}(v) - V_{ij;S}(v) - V_{ij;C}(v) - V_{ij;C'}(v) , \tag{107}$$

and the dots represent higher order terms which are of order $O(1/E^2)$. In the last identity, the integration transformation

$$\int \frac{d^4 l}{(2\pi)^4} \phi^{M_2}(l) \rightarrow \int_0^1 dv \phi^{M_2}(v)$$

has been performed. Up-to twist-3 order, the spin factorization can be obtained by using the identity

$$\phi^{M_2}(v) = -\frac{if_{M_2}}{4N_c}[\gamma_5 \not{v} \phi_P^{M_2}(v) + \mu_\chi^{M_2}(\gamma_5 \hat{\phi}_P^{M_2}(v) - \frac{1}{2}\epsilon_\perp \cdot \sigma \hat{\phi}_\sigma^{M_2}(v))] , \quad (108)$$

where $\epsilon_\perp \cdot \sigma = \epsilon_\perp^{\alpha\beta} \sigma_{\alpha\beta}$, and $\epsilon_\perp^{\alpha\beta} = \epsilon^{\alpha\beta\eta\gamma} n_\eta \bar{n}_\gamma$. The remaining task is to complete the one loop momentum integration over k by using the naive dimensional regularization. The final expression will be given in Section V.

H. Collinear expansion for corrections from penguin contractions

The summation of the contributions from the two diagrams in Fig. 3 (e) and (f) contain the following expression

$$F_j^{B \rightarrow M_1}(q^2) \int \frac{d^4 l}{(2\pi)^4} \text{Tr}[P_{ij}(l) \phi^{M_2}(l)] \quad (109)$$

where the function $P_{ij}(l)$ is defined to contain the quark loop

$$\begin{aligned} & \text{Tr}[P_{ij}(l) \phi^{M_2}(l)] \\ &= i \frac{C_F \pi \alpha_s}{N_c} \int \frac{d^4 k}{(2\pi)^4} \times \left\{ \text{Tr}[\bar{\Gamma}_j \Gamma_i \frac{\not{k} + m_q}{k^2 - m_q^2} \gamma^\alpha \frac{\not{k} + \not{p} + \not{l} + m_q}{(k + p + l)^2 - m_q^2} \bar{\Gamma}_i \phi^{M_2}(l) \gamma^\alpha] \right. \\ &+ \left. \text{Tr}[\bar{\Gamma}_j \Gamma_i \phi^{M_2}(l) \gamma^\alpha] \text{Tr}[\frac{\not{k} + m_q}{k^2 - m_q^2} \gamma^\alpha \frac{\not{k} + \not{p} + \not{l} + m_q}{(k + p + l)^2 - m_q^2} \bar{\Gamma}_i] \right\} \frac{1}{(p + l)^2} . \end{aligned} \quad (110)$$

The internal quark loop momentum k and the internal quark mass m_q has been used in the above expression. To extract the leading contributions, we perform the collinear expansion for $P_{ij}(l)$ in the following way

$$P_{ij}(l) = P_{ij}(\hat{l}) + \frac{\partial P_{ij}}{\partial l^\beta} \Big|_{l=\hat{l}} (l - \hat{l})^\beta + \dots . \quad (111)$$

Due the momentum derivative over l , the second term in the right hand side of the above equation is suppressed by a factor of $O(1/E)$ than the first term. We first concentrate on the first term $P_{ij}(\hat{l})$. Because both the recoil and emitted mesons are energetic, the radiative gluon's momentum scales as $(p + l_2)^\mu \sim (E, E, 0)$. The infrared structure of $P_{ij}(\hat{l})$ then depends on the quark loop integration over k , in which k could be soft or hard. If k is soft, the numerator of the integrand of the k integration decreases one power of k^2 . With

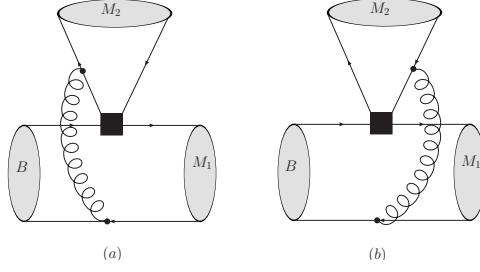


FIG. 5: The hard spectator diagrams.

respect to the loop integration over hard k , the loop integration over soft k is suppressed as $O(1/E^2)$. The hard k region is dominated. The integral transformation is made to transform $\phi^{M_2}(l)$ into $\phi^{M_2}(v)$. The k integration inside $P_{ij}(v)$ is calculated by using NDR. The explicit expression for the above penguin corrections is given in Section V.

I. Collinear expansion for hard spectator corrections

The relevant expression for the spectator diagrams depicted in Fig. 5 is written as

$$\int \frac{d^4 l_B}{(2\pi)^4} \int \frac{d^4 l_1}{(2\pi)^4} \int \frac{d^4 l_2}{(2\pi)^4} \text{Tr}[H(l_B, l_1, l_2) \phi_B(l_B) \phi^{M_1}(l_1) \phi^{M_2}(l_2)] , \quad (112)$$

where the spectator scattering function $H(l_B, l_1, l_2)$ is expressed as

$$\begin{aligned} & \text{Tr}[H(l_B, l_1, l_2) \phi_B(l_B) \phi^{M_1}(l_1) \phi^{M_2}(l_2)] \\ &= -4 \frac{\pi \alpha_s C_F}{N_c^2} \text{Tr}[\Gamma_i \phi_B(l_B) \gamma_\alpha \phi^{M_1}(l_1)] \\ & \quad \times \text{Tr}\left[\left(\gamma^\alpha \frac{\not{l}_2 + \not{k}}{(l_2 + k)^2} \bar{\Gamma}_i - \bar{\Gamma}_i \frac{\not{\bar{l}}_2 + \not{k}}{(\bar{l}_2 + k)^2} \gamma^\alpha\right) \phi^{M_2}(l_2)\right] \frac{1}{k^2} \end{aligned} \quad (113)$$

where $k = l_B - l_1$. We first perform the collinear expansion of $H(l_B, l_1, l_2)$ with respect to the collinear momenta $\hat{l}_i, i = B, 1, 2$,

$$H(l_B, l_1, l_2) = H(\hat{l}_B, \hat{l}_1, \hat{l}_2) + \sum_{i=1,2} \frac{\partial H^{(1)}}{\partial l_i^\nu} \Big|_{l_i=\hat{l}_i} (l_i - \hat{l}_i)^\nu + \dots . \quad (114)$$

Since the M_1 and M_2 mesons are energetic and the spectator quark of the B meson can only carry soft momentum, the virtual gluon momentum k can be soft or hard-collinear. For soft

k , the second trace term vanishes

$$\text{Tr}[(\frac{q^\alpha}{2q \cdot k} \bar{\Gamma}_i - \bar{\Gamma}_i \frac{q^\alpha}{q \cdot k}) \phi^{M_2}(l_2)] \simeq 0 + O(1/E^2), \quad (115)$$

where the errors are estimated to be of $O(1/E^2)$. This shows that the hard spectator diagram contributions are free from infrared divergences up-to $O(1/E)$. However, it has been noted in the literature that there exist end point divergences X_H at $O(1/E)$ as mentioned in Introduction. The term X_H becomes divergent because the pseudoscalar LCDA $\phi_p^{M_1}(u)$ is a constant. Since the constant model for $\phi_p^{M_1}(u)$ is determined by the equation of motion Eq. (4), as we have shown in Section II, the end point divergences can be identified as a mixing effect between the twist-3 and twist-4 LCDAs for the M_1 meson. In addition, we also showed in Section II that, in the energetic meson limit, the pseudoscalar LCDA $\phi_p^{M_1}(u)$ is reduced to be $\hat{\phi}_p^{M_1}(u)$, which is no longer a constant according to the reduced equation of motion Eq. (5). The divergence in X_H as $u \rightarrow 0$ is then regularized by the $\hat{\phi}_p^{M_1}(u)$. The explicit expression for this fact will be given in Section V.

We now show that the $O(1/E)$ contributions can only come from the two parton twist-3 LCDAs for the M_1 meson, and the similar contributions from the two parton twist-3 LCDAs for the M_2 meson are vanishing at $O(1/E)$. The γ^α in the second trace term in Eq. (113) can be $\gamma^\alpha = \not{n} \bar{n}^\alpha$, $\not{n} n^\alpha$, or, $d_\alpha^{\alpha'} \gamma^{\alpha'}$. For the spin state of M_2 being γ_5 , the $\Gamma_i \otimes \bar{\Gamma}_i$ can only be $-2(S-P)(S+P)$. For $H^{(1)}(\hat{l}_B, \hat{l}_1, \hat{l}_2)$, the dominant contributions in the second trace term can only come from $\hat{l}_2 - \hat{l}_1$, or, $\hat{l}_2 - \hat{l}_1$, which are proportional to \not{n} or \not{n} . This selects $\gamma^\alpha = \not{n} \bar{n}^\alpha$, or $\not{n} n^\alpha$. For $\gamma^\alpha = \not{n} n^\alpha$, the contraction of n^α with the first trace term in Eq. (113) gives

$$n^\alpha \text{Tr}[\Gamma_i \phi_B(l_B) \gamma_\alpha \phi^{M_1}(l_1)] . \quad (116)$$

The contact of \not{n} with $\phi^{M_1}(l_1)$ extracts one short distance propagator

$$\frac{-i \not{n}}{2\bar{n} \cdot l_1}$$

and a vertex $-i\gamma^\beta$. The result appears as

$$n^\alpha \text{Tr}[\Gamma_i \phi_B(l_B) \gamma_\alpha \frac{-i \not{n}}{2\bar{n} \cdot l_1} (-i\gamma_\beta) \bar{w}_{\beta'}^\beta \phi_\partial^{M_1\beta}(l_1, l_1)] , \quad (117)$$

where $\bar{w}_{\beta'}^\beta = g_{\beta'}^\beta - n^\beta \bar{n}_{\beta'}$. Because the short distance propagator is of $O(1/E)$, the dimension of hard scattering function is then decreased by one order. The related contributions are

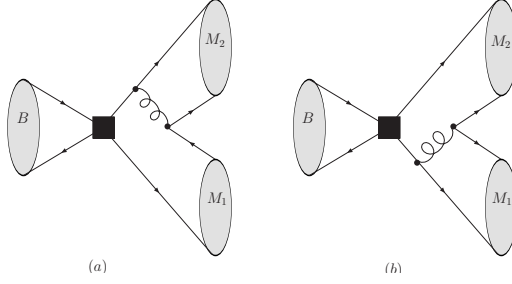


FIG. 6: The diagrams for annihilation corrections from final state emission gluons.

of next twist than what we have considered. The other possibility is that $\gamma^\alpha = \not{n}\bar{n}^\alpha$, which then selects the \hat{y}_2 , or $\hat{\bar{y}}_2$ parts of the propagators in the second trace term. The result is proportional to

$$\left(\frac{1}{2n \cdot q} - \frac{1}{2\bar{n} \cdot q} \right) \bar{n}^\alpha = 0 \quad (118)$$

which is obviously vanishing.

For the spin state of M_2 being $\epsilon_\perp \cdot \sigma$, the $\Gamma_i \otimes \bar{\Gamma}_i$ can only be $-2(S-P)(S+P)$. Similar to the situation for the γ_5 spin state of M_2 , the possible twist-3 contribution can only come from $\gamma^\alpha = \not{n}\bar{n}^\alpha$, since the μ, ν indices of $\sigma^{\mu\nu}$ of $\epsilon_\perp \cdot \sigma$ are transversal. The second trace term is then proportional to

$$\left(\frac{1}{2n \cdot q} - \frac{1}{2\bar{n} \cdot q} \right) \epsilon_\perp^{\mu\nu} \epsilon_{\perp, \mu\nu} = 0 \quad (119)$$

which is also vanishing.

J. Collinear expansion for annihilation corrections from final state emission gluons

The relevant term for the annihilation diagrams depicted in Fig. 6, in which the gluons are emitted from the final state mesons, is written as,

$$\text{Tr}[\phi_B(l_B)\Gamma_i][A_j^f(l_1, l_2)\phi^{M_1}(l_1)\phi^{M_2}(l_2)] \quad (120)$$

where

$$\begin{aligned} & \text{Tr}[A_j^f(l_1, l_2)\phi^{M_1}(l_1)\phi^{M_2}(l_2)] \\ &= 4 \frac{\pi\alpha_s C_F}{N_c^2} \text{Tr}[(\gamma_\alpha \frac{(\not{l}_2 - \not{k})}{(l_2 - k)^2} \bar{\Gamma}_i - \bar{\Gamma}_i \frac{(\not{l}_1 - \not{k})}{(l_1 - k)^2} \gamma_\alpha) \phi^{M_1}(l_1) \gamma^\alpha \phi^{M_2}(l_2)] \frac{1}{k^2} \end{aligned} \quad (121)$$

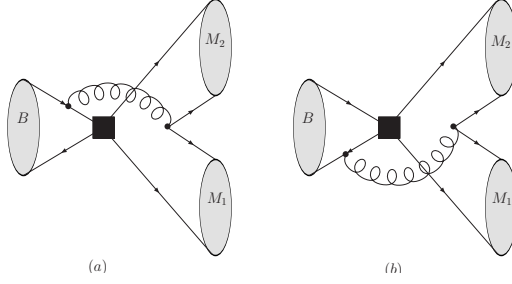


FIG. 7: The diagrams for annihilation corrections from initial state emission gluons.

with $k = \bar{l}_1 - l_2$. The collinear expansion of $A_j^f(l_1, l_2)$ is

$$A_j^f(l_1, l_2) = A_j^f(\hat{l}_1, \hat{l}_2) + \dots, \quad (122)$$

where dots denote the terms of higher than twist-3. Since M_1 and M_2 are assumed to move in opposite directions and carry energetic momenta, the gluon momentum k can be hard or soft. For soft k , the following vanishes

$$\text{Tr}[(\frac{q_\alpha}{q \cdot k} \bar{\Gamma}_i - \bar{\Gamma}_i \frac{p_\alpha}{p \cdot k}) \phi^{M_1}(l_1) \gamma^\alpha \phi^{M_2}(l_2)] \simeq 0 + O(1/E^2). \quad (123)$$

This is because

$$\phi^{M_1}(l_2) \not{k} \phi^{M_2}(l_2) = \phi^{M_1}(l_1) \not{p} \phi^{M_2}(l_2) \simeq 0 + O(1/E^2), \quad (124)$$

where we have used the property of the long distance part of the parton propagators.

K. Collinear expansion for annihilation corrections from initial state emission gluons

The relevant expression for the annihilation diagrams as depicted in Fig. 7 is given by

$$\begin{aligned} & \text{Tr}[A_j^i(l_B, l_1, l_2) \phi_B(l_B) \phi^{M_1}(l_1) \phi^{M_2}(l_2)] \\ &= -2 \frac{\pi \alpha_s C_F}{N_c^2} \text{Tr}[(\gamma_\alpha \frac{(\bar{l}_B + k) + m_b}{(\bar{l}_B + k)^2 - m_b^2} \Gamma_j - \Gamma_j \frac{(\bar{l}_B + k)}{(\bar{l}_B + k)^2} \gamma_\alpha) \phi_B(l_B)] \\ & \quad \text{Tr}[\bar{\Gamma}_j \phi^{M_1}(l_1) \gamma^\alpha \phi^{M_2}(l_2)] \frac{1}{k^2} \end{aligned} \quad (125)$$

with $k = \bar{l}_1 - l_2$. The collinear expansion for $A_j^i(l_B, l_1, l_2)$ with respect to $\hat{l}_{B,1,2}$ is given by

$$A_j^i(l_B, l_1, l_2) = A_j^i(\hat{l}_B, \hat{l}_1, \hat{l}_2) + \dots, \quad (126)$$

where dots denote terms of higher than twist-3. The \hat{l}_B is defined as $\hat{l}_B = \xi P_B$ and the ξ is identified as the momentum fraction carried by the spectator anti-quark of the B meson. Some of order $O(\Lambda^2/E^2)$ terms have been added into the propagator of the spectator anti-quark in $A_j^i(\hat{l}_B, \hat{l}_1, \hat{l}_2)$. This makes the spectator anti-quark be slightly off-shell $\hat{l}_B^2 \simeq \xi^2 m_b^2 \propto \Lambda^2/E^2$. However, for $(V-A)(V \pm A)$ and $-2(S-P)(S+P)$ operators, the final expression for propagator of the spectator anti-quark appears proportional to $1/(\bar{u} - \xi)$ after we have cancelled some common factors of the numerator and denominator of the propagator. This looks like that the spectator anti-quark carries a collinear momentum $\xi E n^\mu$. The subscript j in $A_j^{i,f}$ mean that $j = 1$ for $(V-A)(V-A)$ operators, $j = 2$ for $(V-A)(V+A)$ operators, and $j = 3$ for $-2(S-P)(S+P)$ operators. The superscript i and f in $A_j^{i,f}$ mean that gluon emissions start from the initial or final state. The convention used here is that the M_1 contains an anti-quark from the weak vertex. The anti-quark carries longitudinal momentum fraction as \bar{u} . The M_2 contains a quark from the weak vertex with momentum fraction v . Because the weak annihilation terms are power suppressed than the leading hard spectator interactions by a relative factor $1/m_b$, we identify the weak annihilation correction as subleading corrections. Within our precisions, only terms from twist-2 DAs for the light mesons are considered. Since M_1 and M_2 are assumed to move in opposite directions and carry energetic momenta, the gluon momentum k can be hard or soft. For soft k , the following contraction vanishes

$$\text{Tr}[(\frac{P_{b,\alpha}}{P_b \cdot k} \bar{\Gamma}_i - \bar{\Gamma}_i \frac{P_{b,\alpha}}{P_b \cdot k}) \phi_B(l_B)] \simeq 0 + O(1/E^2) \quad (127)$$

The hard k contributions should give finite results. However, this requires some cares. The second term in the first trace term of $A_j^i(\hat{l}_B, \hat{l}_1, \hat{l}_2)$ is proportional to $(\bar{u} - \xi)^{-1}$. The conventional approach is to neglect the ξ as the authors of [17] has done. This then results in an end-point divergence X_A as $\bar{u} \rightarrow 0$. To solve this divergent problem, we propose to retain the ξ dependence in $A_j^i(\hat{l}_B, \hat{l}_1, \hat{l}_2)$. By introducing a model for $\phi_B(\xi)$, the divergent term X_A appears as

$$X_A \rightarrow \int_0^1 d\xi \phi_B(\xi) \int_0^1 \frac{du}{(\bar{u} - \xi)\bar{u}} \phi_P^{M_1}(u) . \quad (128)$$

IV. COMPARISONS WITH OTHER EXPANSION SCHEMES

In this section, a comparison between different calculation schemes for the contributions involving the pseudotensor LCDA of the final state light mesons will be given.

A. BN scheme

The calculation scheme proposed by Beneke and Neubert in [17, 20] is reviewed below for comparison. We denote this calculation scheme as the BN scheme. Let $H(u, k_\perp, \dots)$ represent any hard scattering function in the amplitude

$$\int_0^1 du \cdots \text{Tr}[H(u, k_\perp, \dots) \Phi^M(u, k_\perp)] , \quad (129)$$

such as the one loop vertex function V , the one loop penguin function P , the hard spectator function H . The twist order of the annihilation contributions involving the twist-3 pseudotensor LCDA of one of final state light mesons in the $B \rightarrow M_1 M_2$ decays are identified as twist-4, they will be neglected in this comparison. The variable u and k_\perp in $H(u, k_\perp, \dots)$ denote the momentum fraction and transverse momenta carried by the partons, respectively. The momentum carried by the quark q is denoted by k_q and the momentum carried by the anti-quark \bar{q} is denoted as $k_{\bar{q}}$. We write k_q and $k_{\bar{q}}$ as

$$\begin{aligned} k_q^\mu &= uEn^\mu + \frac{k_\perp^2}{2uE}\bar{n}^\mu + k_\perp^\mu \\ k_{\bar{q}}^\mu &= uE\bar{n}^\mu + \frac{k_\perp^2}{2\bar{u}E}\bar{n}^\mu - k_\perp^\mu . \end{aligned} \quad (130)$$

The momentum fraction u is defined as $u = n \cdot k_q / n \cdot P$ with P being the meson's momentum as defined in Eq. (7). The $\bar{u} = 1 - u$ is defined as $\bar{u} = n \cdot k_{\bar{q}} / n \cdot P$. The meson state $\Phi^M(u, k_\perp)$ is defined as

$$\Phi^M(u, k_\perp) = -\frac{if_P}{4} \left[\mu_P \gamma_5 \left(\phi_p(u) - i\sigma_{\alpha\beta} \bar{n}^\alpha n^\beta \frac{\phi'_\sigma(u)}{6} + i\sigma_{\alpha\beta} E \bar{n}^\alpha \frac{\phi_\sigma(u)}{6} \frac{\partial}{\partial k_{\perp\beta}} \right) \right] \quad (131)$$

where $\phi_P^M(u)$, $\phi_p^M(u)$, and $\phi_\sigma^M(u)$ are the twist-2 and twist-3 LCDAs for the pseudoscalar meson M , and $\phi_\sigma^{M'}(u) = d\phi_\sigma(u)/du$. The $\partial/\partial k_{\perp\beta}$ in last term of $\Phi^M(u, k_\perp)$ is defined to be applied on the $H(u, k_\perp, \dots)$. After the $\partial/\partial k_{\perp\beta}$ has been applied on $H(u, k_\perp, \dots)$, a collinear limit $k_\perp \rightarrow 0$ is followed to be applied [17, 20]. Because $H(u, k_\perp, \dots)$ can be expanded in

k_\perp , one can use the following transformation [17, 20]

$$\frac{\partial}{\partial k_{\perp\beta}} \rightarrow \frac{2k_\perp^\beta}{k_\perp^2} \quad (132)$$

to simplify the calculation

$$\frac{\partial}{\partial k_{\perp\beta}} k_\perp^\lambda \rightarrow \frac{\langle k_\perp^\beta k_\perp^\lambda \rangle}{k_\perp^2} = d_\perp^{\beta\lambda}, \quad (133)$$

where $d_\perp^{\beta\lambda} = g^{\beta\lambda} - \bar{n}^\beta n^\lambda - \bar{n}^\lambda n^\beta$ and the bracket in $\langle k_\perp^\beta k_\perp^\lambda \rangle$ represents an average over the azimuthal angle in the integration over k_\perp .

The next step is to use the equations of motion, Eq. (4), to rewrite the twist-3 part of $\Phi^M(u, k_\perp)$ in the following expression [17, 20]

$$\Phi^M(u, k_\perp, \dots) \Big|_{tw3} = -\frac{if_P \mu_\chi^M}{4} \gamma_5 \frac{k_q k_{\bar{q}}}{k_q \cdot k_{\bar{q}}} \phi_p(u). \quad (134)$$

We note that some terms of $O(\Lambda/E)$ have been added to arrive at the above compact form. In addition, the solution based on Eq. (4) $\phi_p^M(u) = 1$ and $\phi_\sigma^M(u) = 6u\bar{u}$ have been used to derive Eq. (134). Finally, the Eq. (134) is substituted into Eq. (129) to arrive at

$$-\frac{if_P \mu_P}{4} \int_0^1 du \dots \text{Tr}[H(u, k_\perp, \dots) \gamma_5 \frac{k_q k_{\bar{q}}}{k_q \cdot k_{\bar{q}}}] \Big|_{k_\perp \rightarrow 0} \phi_p(u). \quad (135)$$

For comparison, we suggest to use the following expression

$$\begin{aligned} & -\frac{if_P \mu_P}{4} \int_0^1 du \dots [\text{Tr}[H(u, \dots) \gamma_5] \phi_p(u) \\ & \quad - i \text{Tr}[H(u, \dots) \sigma_{\alpha\beta} \gamma_5] \bar{n}^\alpha n^\beta \frac{\phi'_\sigma(u)}{6} \\ & \quad + i \text{Tr}[\frac{\partial H(u, k_{q\perp}, \dots)}{\partial k_{q\perp}^\beta} \sigma_{\alpha\beta} \gamma_5] \Big|_{k_{q\perp}=0} E \bar{n}^\alpha \frac{\phi_\sigma(u)}{6}] . \end{aligned} \quad (136)$$

Using this calculation scheme, the authors of [17, 20] found that the relevant vertex contributions and penguin contributions are IR finite, while the hard scattering contributions contain the divergent term X_H .

B. DYZ scheme

Du *et. al.* [23, 24] proposed to perform the calculations (such as the Dirac algebra and the integration over the virtual gluon's momentum) for the contributions involving $\phi_\sigma^M(u)$

in the coordinate space. After the calculations have been completed, the expression is then transformed into the momentum space. For comparison, we write the resultant expression in the momentum space as

$$-\frac{if_{P\mu P}}{4}\int_0^1 du \cdots \left[\text{Tr}[H(u, \cdots)\gamma_5]\phi_p(u) + i\text{Tr}\left[\frac{\partial H(l, \cdots)}{\partial l^\beta}\sigma^{\alpha\beta}\gamma_5\right]|_{l=l}E\bar{n}_\alpha\frac{\phi_\sigma(u)}{6} \right] \quad (137)$$

Using this calculation scheme, Du *et. al.* found that it is necessary to require a symmetric criteria for the $\phi_\sigma(u)$ to regularize the infrared divergences from the vertex and penguin contributions. However, even requiring the symmetric criteria, the IR divergences still exist in the hard spectator contributions.

C. CE scheme

Based on the analysis given in Section III, the relevant amplitudes for one loop contributions with a pseudotensor LCDA calculated under the CE scheme can be formally written as

$$\begin{aligned} & -\frac{if_{P\mu P}}{4}\int_0^1 du \cdots \left[\text{Tr}[H(u, \cdots)\gamma_5]\hat{\phi}_p(u) - \frac{1}{2}\text{Tr}[H(u, \cdots)\epsilon_\perp^{\alpha\beta}\sigma_{\alpha\beta}]\hat{\phi}_\sigma(u) \right] \\ & + \frac{1}{8}\int_0^1 du \cdots \text{Tr}[H_\mu(u, u, \cdots)\sigma_{\alpha\beta}\gamma_5]w_{\mu'}^\mu\phi_\partial^{\alpha\beta\mu'}(u, u) , \end{aligned} \quad (138)$$

where

$$\phi_\partial^{\alpha\beta\mu'}(u, u) = \int_0^\infty \frac{d\lambda}{2\pi} e^{-i\bar{u}\lambda} \langle M|\bar{q}(0)\sigma^{\alpha\beta}\gamma_5 i\partial^{\mu'}(\lambda n/E)q(\lambda n/E)|0\rangle . \quad (139)$$

Under the CE scheme, the analysis given in Section III and the explicit expressions for the relevant one loop contributions in the next Section, show that the vertex, penguin and hard scattering contributions are IR finite up-to twist-3 under the two particle approximation. The last term in Eq. (138) is of higher twist than three. We retain it in Eq. (138) is only for comparisons with the other schemes.

D. Comparisons

We now summarize the differences between these three calculation schemes.

1. *The interpretation of the derivative hard function*

The derivative functions $\partial H(u, l_\perp, \dots)/\partial l_\perp^\beta$ in the BN scheme or $\partial H(u, l, \dots)/\partial l^\beta$ in the DYZ schemes are interpreted as short distance hard scattering functions related to $\phi_\sigma(u)$. The derivative $\partial/\partial l_\perp^\beta$ in the BN scheme, or $\partial/\partial l^\beta$ in the DYZ scheme come from the coordinate variable in the spin projector associated with the $\phi_\sigma(u)$. In the CE scheme, the derivative function $\partial H(u, l, \dots)/\partial l^\mu$ arises from a Taylor expansion of the H with respect to the collinear momentum \hat{l} . By a simple manipulation, we can write the following corresponding relations between the terms calculated in the BN and the CE schemes as

$$\begin{aligned} \text{Tr}[H(u, \dots)\sigma_{\alpha\beta}\gamma_5\bar{n}^\alpha n^\beta]^{BN} \frac{\phi'_\sigma(u)}{3} &\leftrightarrow -i\text{Tr}[H(u, \dots)\epsilon_\perp^{\alpha\beta}\sigma_{\alpha\beta}]^{CE} \hat{\phi}_\sigma(u) , \\ \text{Tr}\left[\frac{\partial H(l, \dots)}{\partial l_\perp^\beta}\sigma^{\alpha\beta}\gamma_5\right]^{BN}|_{l=\hat{l}} E\bar{n}_\alpha f_{P\mu P} \frac{\phi_\sigma(u)}{3} &\leftrightarrow -i\text{Tr}[H_\mu(u, u, \dots)\sigma_{\alpha\beta}\gamma_5]^{CE} d_{\mu'}^\mu \phi_\sigma^{\alpha\beta\mu'}(u, u) . \end{aligned} \quad (140)$$

Similarly, we can also obtain the following relationships between terms derived by the DYZ scheme and the CE scheme as

$$\begin{aligned} \text{Tr}\left[\frac{H(l, \dots)}{\partial l^\beta}\sigma^{\alpha\beta}\right]_{l=\hat{l}}^{DYZ} E\bar{n}_\alpha f_{P\mu P} \frac{\phi_\sigma(u)}{3} &\leftrightarrow i\text{Tr}[H(u, \dots)\epsilon_\perp^{\alpha\beta}\sigma_{\alpha\beta}]^{CE} f_{P\mu P} \hat{\phi}_\sigma(u) \\ &\quad -i\text{Tr}[H_\mu(u, u, \dots)\sigma_{\alpha\beta}\gamma_5]^{CE} d_{\mu'}^\mu \phi_\sigma^{\alpha\beta\mu'}(u, u) . \end{aligned} \quad (141)$$

In the above expressions, we have assumed that $\phi'_\sigma(u)$ and $\phi_\sigma(u)$ are scheme independent. The BN, DYZ, and CE superscripts in the hard scattering functions represent the functions having been calculated under the BN scheme, the DYZ scheme, and the CE scheme, respectively. Under the two particle approximation, $H_\mu(u, u, \dots)$ is identical to $(\partial H/\partial l^\mu)_{l=\hat{l}}$. And, $\partial H(l, \dots)/\partial l^\mu$ is $1/E$ suppressed than $H(l, \dots)$. The main differences between the CE scheme and the other two schemes are from the spin projector $i\epsilon^{\alpha\beta\nu\mu}En_\alpha$ introduced in the BN and DYZ schemes. The E factor from the spin projector has the effects to increase one order in the dimension of $\partial H(l, \dots)/\partial l^\mu$. Therefore, the product of $\partial H(l, \dots)/\partial l^\mu$ and $i\epsilon^{\alpha\beta\nu\mu}En_\alpha$ becomes of the same order as of order of $H(l, \dots)$. On the other hand, in the CE scheme, the $\partial H(l, \dots)/\partial l^\mu$ is not assumed to be associated with a large factor E .

2. The twist identification

The analysis made in Section II indicates that the $\phi_\sigma(u)$ (or $\phi_\sigma^\perp(u)$) should be identified as one twist order higher than the twist order of $\phi'_\sigma(u)$ (or $\phi_\sigma^\parallel(u)$) under the energetic light meson limit. Namely, $\phi_\sigma(u)$ is of $O(\Lambda/E)$ at $u = O(1)$ and $\phi'_\sigma(u) = O(1)$ at $u = O(1)$. The term

$$i\text{Tr}\left[\frac{\partial H(l, \dots)}{\partial l_\perp^\beta} \sigma^{\alpha\beta}\right]^{(BN)} \Big|_{l=\hat{l}} E \bar{n}_\alpha \frac{\phi_\sigma(u)}{6}$$

is $1/E$ suppressed than the term

$$-i\text{Tr}[H(u, \dots) \sigma_{\alpha\beta} \gamma_5 \bar{n}^\alpha n^\beta]^{(BN)} \frac{\phi'_\sigma(u)}{6}.$$

Thus, the calculations made by the BN scheme under the energetic meson limit become consistent with the results obtained by using the CE scheme. This fact can be seen in the next Section, where the explicit results for the one loop contributions calculated by the CE scheme will be given. Unlike the BN and CE schemes, the identification of different twist order is unclear in the DYZ scheme. The contributions of different order of magnitudes are mixing together.

V. TWIST-3 TWO PARTICLE CONTRIBUTIONS IN $B \rightarrow \pi K$ DECAYS

The predictions for the penguin dominated B decay processes under QCDF approach are related to the $X_{H,A}$ terms. We consider the $B \rightarrow \pi K$ decays as an example to illustrate how our results obtained in previous sections can be used to improve our understandings for these decays. The matrix elements of the effective weak Hamiltonian can be written as, up to $O(1/m_b)$, in the convention of [18]

$$\langle \pi K | H_{eff} | \bar{B} \rangle = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \langle \pi K | T_p + T_p^{ann} | \bar{B} \rangle \quad (142)$$

where

$$\begin{aligned} T_p = & a_1(\pi K) \delta_{pu} (\bar{u}b)_{V-A} \otimes (\bar{s}u)_{V-A} + a_2(\pi K) \delta_{pu} (\bar{s}b)_{V-A} \otimes (\bar{u}u)_{V-A} \\ & + a_3(\pi K) \sum_q (\bar{s}b)_{V-A} \otimes (\bar{q}q)_{V-A} + a_4^p(\pi K) \sum_q (\bar{q}b)_{V-A} \otimes (\bar{s}q)_{V-A} \\ & + a_5(\pi K) \sum_q (\bar{s}b)_{V-A} \otimes (\bar{q}q)_{V+A} + a_6(\pi K) \sum_q (-2)(\bar{q}b)_{S-P} \otimes (\bar{s}q)_{S+P} \end{aligned}$$

$$\begin{aligned}
& +a_7(\pi K) \sum_q (\bar{s}b)_{V-A} \otimes \frac{3}{2} e_q(\bar{q}q)_{V+A} \\
& +a_8(\pi K) \sum_q (-2)(\bar{q}b)_{S-P} \otimes \frac{3}{2} e_q(\bar{s}q)_{S+P} \\
& +a_9(\pi K) \sum_q (\bar{s}b)_{V-A} \otimes \frac{3}{2} e_q(\bar{q}q)_{V-A} \\
& +a_{10}(\pi K) \sum_q (\bar{q}b)_{V-A} \otimes \frac{3}{2} e_q(\bar{s}q)_{V-A} ,
\end{aligned} \tag{143}$$

where $(\bar{q}_1 q_2)_{V\pm A} = \bar{q}_1 \gamma^\mu (1 \pm \gamma_5) q_2$ and $(\bar{q}_1 q_2)_{S\pm P} = \bar{q}_1 (1 \pm \gamma_5) q_2$. The symbol \otimes in T_p implies that the matrix elements of the operators in T_p are evaluated according to the factorized form $\langle \pi K | j_1 \otimes j_2 | \bar{B} \rangle \equiv \langle \pi | j_1 | \bar{B} \rangle \langle K | j_2 | 0 \rangle$ or $\langle K | j_1 | \bar{B} \rangle \langle \pi | j_2 | 0 \rangle$. The contributions relate to T_p^{ann} arise from the weak annihilation contributions with a set of coefficients $b_i(\pi K)$.

The expressions for $a_i(\pi K)$ are written as [18, 20]

$$\begin{aligned}
a_i^p(M_1 M_2) &= (C_i + \frac{C_{i\pm 1}}{N_c}) N_i(M_2) \\
&+ \frac{C_{i\pm 1}}{N_c} \frac{\alpha_s C_F}{4\pi} [V_i(M_2) + \frac{4\pi^2}{N_c} H_i(M_1 M_2)] + P_i^p(M_2) ,
\end{aligned} \tag{144}$$

where the upper or lower signs correspond to the odd i or even i . The superscript p is only used for $i \geq 3$. The leading order coefficients $N_i(M_2)$ represent the normalization integral of the distribution amplitude $\phi_P^{M_2}$ or $\hat{\phi}_p^{M_2}$

$$N_i(M_2) = \begin{cases} 0; & i = 6, 8 \\ 1; & i = 1 - 5, 7, 9, 10 \end{cases} \tag{145}$$

The vertex corrections are written as

$$V_i(M_2) = \begin{cases} \int_0^1 du \phi_P^{M_2}(u) \left[12 \ln \left(\frac{\mu}{m_b} \right) - 18 + g(u) \right]; & i = 1 - 4, 9, 10 \\ \int_0^1 du \phi_P^{M_2}(u) \left[-12 \ln \left(\frac{\mu}{m_b} \right) + 6 - g(1-u) \right]; & i = 5, 7 \\ \int_0^1 du \hat{\phi}_p^{M_2}(u) [-6 + h(u)]; & i = 6, 8 \end{cases} \tag{146}$$

where

$$\begin{aligned}
g(v) &= 3 \left(\frac{1-2u}{1-u} \ln u - i\pi \right) \\
&+ \left[2Li_2(u) - \ln^2 u + \frac{2 \ln u}{1-u} - (3 + 2i\pi) \ln u - (u \leftrightarrow (1-u)) \right] \\
h(v) &= 2 [Li_2(u) - (1 + i\pi) \ln u - (u \leftrightarrow (1-u))] ,
\end{aligned} \tag{147}$$

where we have employed the naive dimensional regularization (NDR) scheme with an anti-commuting γ_5 for the regularization of the UV or IR divergences arising from the loop corrections. The calculations for $V_i(M_2)$, $i = 1 - 5, 7, 9, 10$ have been checked by using the CE scheme. The results have been found to be consistent with the results derived by using the BN scheme. This is reasonable because CE and BN schemes are equivalent at the leading twist order. We also note that the $V_{6,8}$ calculated under the CE scheme are also consistent with those obtained by the BN scheme. This is due to the fact that the main contributions in $V_{6,8}(M_2)$ are from the projection onto the $\phi_p^{M_2}$. The detailed calculations for $V_6(M_2)$ under the CE scheme will be given in the Appendix A. The integrations

$$\begin{aligned} \int_0^1 du g(u) \phi_P^{M_2}(u) &= -\frac{1}{2} - 3i\pi, \\ \int_0^1 du h(u) \hat{\phi}_p^{M_2}(u) &= 0, \end{aligned} \quad (148)$$

lead to the values of the vertex corrections at the scale μ as

$$V_i(M_2) = \begin{cases} \left[12 \ln\left(\frac{\mu}{m_b}\right) - \frac{37}{2} - 3i\pi \right]; & i = 1 - 4, 9, 10 \\ \left[-12 \ln\left(\frac{\mu}{m_b}\right) + \frac{13}{2} + 3i\pi \right]; & i = 5, 7 \\ -6; & i = 6, 8 \end{cases} \quad (149)$$

The penguin contributions are given by

$$\begin{aligned} P_4^p(M_2) &= \frac{\alpha_s C_F}{4\pi N_c} \left\{ C_1 \left[\frac{4}{3} \ln \frac{m_b}{\mu} + \frac{2}{3} - G_{M_2}(s_p) \right] + C_3 \left[\frac{8}{3} \ln \frac{m_b}{\mu} + \frac{4}{3} - G_{M_2}(0) - G_{M_2}(1) \right] \right. \\ &\quad \left. + (C_4 + C_6) \left[\frac{4n_f}{3} \ln \frac{m_b}{\mu} - (n_f - 2)G_{M_2}(0) - G_{M_2}(s_c) - G_{M_2}(1) \right] \right. \\ &\quad \left. - 2C_{8g}^{eff} \int_0^1 \frac{du}{1-u} \phi_P^{M_2}(u) \right\}, \\ P_6^p(M_2) &= \frac{\alpha_s C_F}{4\pi N_c} \left\{ C_1 \left[\frac{4}{3} \ln \frac{m_b}{\mu} + \frac{2}{3} - \hat{G}_{M_2}(s_p) \right] + C_3 \left[\frac{8}{3} \ln \frac{m_b}{\mu} + \frac{4}{3} - \hat{G}_{M_2}(0) - \hat{G}_{M_2}(1) \right] \right. \\ &\quad \left. + (C_4 + C_6) \left[\frac{4n_f}{3} \ln \frac{m_b}{\mu} + \frac{2}{3} - (n_f - 2)\hat{G}_{M_2}(0) - \hat{G}_{M_2}(s_c) - \hat{G}_{M_2}(1) \right] \right. \\ &\quad \left. - 2C_{8g}^{eff} \int_0^1 du \hat{\phi}_p^{M_2}(u) \right\}, \\ P_{10}^p(M_2) &= \frac{\alpha}{9\pi N_c} \left\{ (C_1 + N_c C_2) \left[\frac{4}{3} \ln \frac{m_b}{\mu} + \frac{2}{3} - G_{M_2}(s_p) \right] - 3C_{7\gamma}^{eff} \int_0^1 \frac{du}{1-u} \phi_P^{M_2}(u) \right\}, \\ P_8^p(M_2) &= \frac{\alpha}{9\pi N_c} \left\{ (C_1 + N_c C_2) \left[\frac{4}{3} \ln \frac{m_b}{\mu} + \frac{2}{3} - \hat{G}_{M_2}(s_p) \right] - 3C_{7\gamma}^{eff} \int_0^1 du \hat{\phi}_p^{M_2}(u) \right\}, \end{aligned} \quad (150)$$

where $n_f = 5$ denotes the number of light quark flavors. The mass ratios $s_u = 0$ and $s_c = m_c^2/m_b^2$ are introduced. According to the conventions used in [18, 20], the electroweak corrections from C_{7-10} are neglected. The functions $G_{M_2}(s)$ and $\hat{G}_{M_2}(s)$ are defined as [18, 20]

$$\begin{aligned} G_{M_2}(s) &= \int_0^1 du G(s - i\epsilon, 1 - u) \phi_P^{M_2}(u) , \\ \hat{G}_{M_2}(s) &= \int_0^1 du G(s - i\epsilon, 1 - u) \hat{\phi}_p^{M_2}(u) , \\ G(s, u) &= -4 \int_0^1 dx x(1 - x) \ln[s - x(1 - x)u] . \end{aligned} \quad (151)$$

The effective Wilson coefficients are given by $C_{7\gamma}^{eff} = C_{7\gamma} - \frac{1}{3}C_5 - C_6$ and $C_{8g}^{eff} = C_8 + C_5$. Under the energetic limit, we choose the asymptotic form for both $\phi_P^{M_2}(u)$ and $\hat{\phi}_p^{M_2}(u)$ as $\phi_P^{M_2}(u) = \hat{\phi}_p^{M_2}(u) = 6u\bar{u}$. The asymptotic models for $\phi_P^{M_2}(u, \mu)$ and $\hat{\phi}_p^{M_2}(u, \mu)$ are defined as the asymptotic limit $\mu \rightarrow \infty$ for the distribution amplitudes. At finite renormalization scale, the distribution amplitudes are expanded into Gegenbauer polynomials

$$\phi_P^{M_2}(u, \mu) = 6u\bar{u} \left[1 + \sum_{n=1}^{\infty} \alpha_n^{M_2}(\mu) C_n^{3/2}(2u - 1) \right] , \quad (152)$$

where the Gegenbauer moments $\alpha_n^{M_2}(\mu)$ are multiplicatively renormalized. The scale dependence of the Gegenbauer moments $\alpha_n^{M_2}(\mu)$ enters the vertex and penguin corrections at order α_s^2 [18, 20]. In the next-to-leading calculation as we have done in this paper, the Gegenbauer moments can be neglected. In this approximation, we then arrive at a further simplification $G_{M_2}(s) = \hat{G}_{M_2}(s)$. This results in the following identities

$$P_6^p(M_2) = P_4^p(M_2) + \frac{\alpha_s C_F}{\pi N_c} C_{8g}^{eff} , \quad (153)$$

$$P_8^p(M_2) = P_{10}^p(M_2) + \frac{2\alpha}{3\pi N_c} C_{7\gamma}^{eff} . \quad (154)$$

For practical applications, the $G_{M_2}(s)$ are evaluated under the previously mentioned approximations as

$$G_{M_2}(0) = \frac{5}{3} + \frac{2i\pi}{3} , \quad (155)$$

$$G_{M_2}(1) = \frac{85}{3} - 6\sqrt{3} + \frac{4\pi}{9} , \quad (156)$$

$$\begin{aligned} G_{M_2}(s_c) &= \frac{5}{3} - \frac{2}{3} \ln s_c + \frac{32}{3} s_c + 16s_c^2 \\ &\quad - \frac{2}{3} \sqrt{1 - 4s_c} \left[1 + 2s_c + 24s_c^2 \right] (2\text{arctanh}\sqrt{1 - 4s_c} - i\pi) \\ &\quad + 12s_c^2 \left[1 - \frac{4}{3}s_c \right] (2\text{arctanh}\sqrt{1 - 4s_c} - i\pi)^2 + \dots . \end{aligned} \quad (157)$$

The complete expressions for $G_{M_2}(0)$, $G_{M_2}(1)$ and $G_{M_2}(s_c)$ with Gegenbauer moments are referred to [18, 20].

The scale for the vertex and penguin corrections refers to the parton off-shellness in the loop diagrams. The typical setting of the scale is chosen to be $\mu \sim m_b$, which are the scales substituted in the Wilson coefficients C_i and in the hard scattering kernel T^I . The combination of the scale dependences and the renormalization scheme-dependent constants in C_i , $V_i(M_2)$ and $P_i^p(M_2)$ give renormalization-group invariant results.

The hard spectator corrections are given by

$$H_i(M_1 M_2) = \frac{B_{M_1 M_2}}{A_{M_1 M_2}} m_B \int_0^1 d\xi \frac{\phi_B(\xi)}{\xi} \int_0^1 du \int_0^1 dv \left[\frac{\phi_P^{M_1}(u) \phi_P^{M_2}(v)}{\bar{u}(\bar{v} - \xi)} + r_\chi^{M_1} \frac{\hat{\phi}_p^{M_1}(u) \phi_P^{M_2}(v)}{u(\bar{v} - \xi)} \right], \quad (158)$$

for $i = 1 - 4, 9, 10$, and

$$H_i(M_1 M_2) = \frac{B_{M_1 M_2}}{A_{M_1 M_2}} m_B \int_0^1 d\xi \frac{\phi_B(\xi)}{\xi} \int_0^1 du \int_0^1 dv \left[\frac{\phi_P^{M_1}(u) \phi_P^{M_2}(v)}{u(\bar{v} - \xi)} + r_\chi^{M_1} \frac{\hat{\phi}_p^{M_1}(u) \phi_P^{M_2}(v)}{\bar{u}(\bar{v} - \xi)} \right], \quad (159)$$

for $i = 5, 7$, and $H_{6,8}(M_1 M_2) = 0$. Different from the BN scheme, we have introduced the distribution amplitude for the B meson and retained the ξ dependence in the spectator propagator. Since the $\hat{\phi}_p^{M_1}(u) = 6u\bar{u}$ is no longer a constant, there are no end-point divergences as in those results derived in the BN [18, 20] or DYZ schemes [23, 24]. Due to the soft scale associated with the spectator quark, the introduction of the B meson's DA $\phi_B(\xi)$ may need to consider the effects of double logarithms from a overlap of soft and collinear divergences associated with the spectator quark. This may need to introduce a Sudakov form factor to account for the effects of double logarithms. A two loop analysis for the hard spectator made by Beneke and Yang [46] indicates that there exists no double logarithms up-to $O(\alpha_s)$. Therefore, at next-to-leading order, we can neglect the Sudakov form factor completely. By using the model for $\phi_B(\xi)$ and the fact that $\phi_P^{M_1}(u) = \hat{\phi}_p^{M_1}(u)$, we arrive at a simple expression for $H_i(M_1 M_2)$, which is useful for numeric calculations,

$$H_i(M_1 M_2) = \frac{B_{M_1 M_2}}{A_{M_1 M_2}} m_B (1 + r_\chi^{M_1}) N_H, \quad (160)$$

where $N_H = 18/\lambda_B$. The model for $\phi_B(\xi)$ is assumed to have the form

$$\phi_B(\xi) = \frac{N_B \xi^2 \bar{\xi}^2}{[\xi^2 + \epsilon_B \bar{\xi}]^2} \quad (161)$$

with $N_B = 0.133$, $\epsilon_B = 0.005$ for $\lambda_B = 350$ MeV. The N_B and ϵ_B are determined according to the conditions

$$\begin{aligned}\int_0^1 d\xi \phi_B(\xi) &= 1, \\ \int_0^1 d\xi \frac{\phi_B(\xi)}{\xi} &= \frac{m_B}{\lambda_B},\end{aligned}\tag{162}$$

where the errors are controlled within 1%. This is consistent with the conventional assumption $\lambda_B \leq 600$ MeV under the condition $3\lambda_B \leq 4\bar{\Lambda}$ with $\bar{\Lambda} = m_B - m_b$. The scale dependence in $r_\chi^{M_1}(\mu_h)$ is chosen as $\mu_h = \sqrt{\Lambda_h \mu}$ with $\Lambda_h = 0.5$ GeV [18, 20].

The annihilation corrections are expressed in terms of the following $A_j^{i,f}$, $j = 1, \dots, 3$ functions

$$\begin{aligned}A_1^i &= \pi\alpha_s \int_0^1 d\xi \phi_B(\xi) \int_0^1 du \phi_P^{M_2}(u) \int_0^1 dv \phi_P^{M_1}(v) \left[\frac{1}{(1 - (u - \xi)(\bar{\xi} - v))v} + \frac{1}{(\bar{u} - \xi)\bar{u}v} \right], \\ A_2^i &= \pi\alpha_s \int_0^1 d\xi \phi_B(\xi) \int_0^1 du \phi_P^{M_2}(u) \int_0^1 dv \phi_P^{M_1}(v) \left[\frac{1}{(1 - (u - \xi)(\bar{\xi} - v))\bar{u}} + \frac{1}{(v - \xi)\bar{u}v} \right], \\ A_3^i &= A_1^f = A_2^f = A_3^f = 0.\end{aligned}\tag{163}$$

The subscript j in $A_j^{i,f}$ mean that $j = 1$ for $(V - A)(V - A)$ operators, $j = 2$ for $(V - A)(V + A)$ operators, and $j = 3$ for $-2(S - P)(S + P)$ operators. The superscript i and f in $A_j^{i,f}$ mean that gluon emissions start from the initial or final state. The convention used here is that the M_1 contains an anti-quark from the weak vertex. The anti-quark carries longitudinal momentum fraction as \bar{u} . The M_2 contains a quark from the weak vertex with momentum fraction v . Because the weak annihilation terms are power suppressed than the leading hard spectator interactions by a relative factor $1/m_b$, we identify the weak annihilation correction as subleading corrections. Within our precisions, only terms from twist-2 DAs for the light mesons are considered. The non-singlet annihilation coefficients are given by

$$\begin{aligned}b_1 &= \frac{C_F}{N_c^2} C_1 A_1^i, & b_2 &= \frac{C_F}{N_c^2} C_2 A_1^i, \\ b_3^p &= \frac{C_F}{N_c^2} C_3 A_1^i, & b_4^p &= \frac{C_F}{N_c^2} [C_4 A_1^i + C_6 A_2^i], \\ b_{3,EW}^p &= \frac{C_F}{N_c^2} C_9 A_1^i, & b_{4,EW}^p &= \frac{C_F}{N_c^2} [C_{10} A_1^i + C_8 A_2^i].\end{aligned}\tag{164}$$

We note that the coefficients (b_1, b_2) corresponds to the current-current annihilation, (b_3, b_4) corresponds to the penguin annihilation, and $(b_{3,EW}, b_{4,EW})$ to the electroweak penguin annihilation. The end-point divergences as the spectator quark approaching its on-shell are

regularized by the momentum fraction ξ , which is defined as $\hat{l}_B = \xi P_b$. Such a definition for ξ would make the spectator quark slightly off-shell since $\hat{l}_B^2 = \xi^2 P_b^2 \sim \lambda_B^2$, which is of order Λ_{QCD}^2 . The errors from this off-shell-ness for the amplitude is of order $O(\Lambda^2/E^2)$, which is of twist-4. Within our precision, the errors can be neglected.

Using the same approximations for the hard spectator corrections, we arrive at the following simplified expressions for the annihilation corrections

$$A_1^i = A_2^i \simeq 18\pi\alpha_s \left(\frac{\pi^2 - 9}{3} + N_A \right), \quad (165)$$

where $N_A = -12.1 - 1.7i$ is related to the model for $\phi_B(\xi)$. The convolution integrations appear to be overlapped integrals over ξ , u and v with $\phi_B(\xi)$, $\phi_P^{M_1}(u)$ and $\phi_P^{M_2}(v)$. The traditional end point divergences as $u \rightarrow 1$ or $v \rightarrow 1$ are then regularized by the B meson distribution amplitude $\phi_B(\xi)$. Although the averaged value of the ξ is much smaller than one, however, we argue that it still can not be neglected. The scale dependence in $\alpha_s(\mu)$ is chosen as $\mu_h = \sqrt{\Lambda_h \mu}$ with $\Lambda_h = 0.5$ GeV.

For penguin dominant $B \rightarrow \pi K$ decays, the relevant decay amplitudes under QCD factorization are parametrized as the following [18]

$$\begin{aligned} A(B^- \rightarrow \pi^- \bar{K}^0) &= \lambda_p \left[(a_4^p - \frac{1}{2}a_{10}^p) + r_\chi^K (a_6^p - \frac{1}{2}a_8^p) \right] A_{\pi K} \\ &\quad + (\lambda_u b_2 + (\lambda_u + \lambda_c)(b_3 + b_3^{EW})) B_{\pi K}, \\ -\sqrt{2}A(B^- \rightarrow \pi^0 K^-) &= [\lambda_u a_1 + \lambda_p(a_4^p + a_{10}^p) + \lambda_p r_\chi^K (a_6^p + a_8^p)] A_{\pi K} \\ &\quad + [\lambda_u a_2 + (\lambda_u + \lambda_c)\frac{3}{2}(-a_7 + a_9)] A_{K\pi} \\ &\quad + [\lambda_u b_2 + (\lambda_u + \lambda_c)(b_3 + b_3^{EW})] B_{\pi K}, \\ -A(\bar{B}^0 \rightarrow \pi^+ K^-) &= [\lambda_u a_1 + \lambda_p(a_4^p + a_{10}^p) + \lambda_p r_\chi^K (a_6^p + a_8^p)] A_{\pi K} \\ &\quad + [(\lambda_u + \lambda_c)(b_3 - \frac{1}{2}b_3^{EW})] B_{\pi K} \\ \sqrt{2}A(\bar{B}^0 \rightarrow \pi \bar{K}^0) &= A(B^- \rightarrow \pi^- \bar{K}^0) + \sqrt{2}A(B^- \rightarrow \pi^0 K^-) - A(\bar{B}^0 \rightarrow \pi^+ K^-) \end{aligned} \quad (166)$$

where $\lambda_p = V_{pb}V_{ps}^*$, $a_i \equiv a_i(\pi K)$, and $\lambda_p a_i^p = \lambda_u a_i^u + \lambda_c a_i^c$. The CP conjugation of decay amplitudes are obtained by replacing $\lambda_p \rightarrow \lambda_p^*$ for the above amplitudes. The $A_{\pi K}$, $A_{K\pi}$, and $B_{\pi K}$ are defined as

$$\begin{aligned} A_{\pi K} &= i \frac{G_F}{\sqrt{2}} (m_B^2 - m_\pi^2) F_0^{B \rightarrow \pi}(m_K^2) f_K, \\ A_{K\pi} &= i \frac{G_F}{\sqrt{2}} (m_B^2 - m_K^2) F_0^{B \rightarrow K}(m_\pi^2) f_\pi, \end{aligned}$$

$$B_{\pi K} = i \frac{G_F}{\sqrt{2}} f_B f_K f_\pi . \quad (167)$$

For numerical calculations, we will use the following input parameters

$$\begin{aligned} \Lambda_{\overline{MS}}^{(5)} &= 0.225 \text{GeV} , \quad m_b(m_b) = 4.2 \text{GeV} \quad m_c(m_b) = 1.3 \text{GeV} , \quad m_s(2 \text{GeV}) = 0.090 \text{GeV} , \\ |V_{cb}| &= 0.41 , \quad |V_{ub}/V_{cb}| = 0.09 , \quad \gamma = 70^\circ , \quad \tau(B^-) = 1.67 \text{(ps)} , \\ \tau(B_d) &= 1.54 \text{(ps)} , \quad f_\pi = 131 \text{MeV} , \quad f_K = 160 \text{MeV} , \quad f_B = 200 \text{MeV} , \\ F_0^{B \rightarrow \pi} &= 0.28 , \quad F_0^{B \rightarrow K} = 0.34 . \end{aligned} \quad (168)$$

For λ_u and λ_c , we take the following convention for their parametrization

$$\frac{\lambda_u}{\lambda_c} = \tan^2 \theta_c R_b e^{-i\gamma} \quad (169)$$

where

$$\begin{aligned} \tan^2 \theta_c &= \frac{\lambda^2}{1 - \lambda^2} , \\ R_b &= \frac{1 - \lambda^2/2}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| , \\ \lambda &= |V_{us}| . \end{aligned} \quad (170)$$

The value of λ is taken as 0.22.

The values of the NLO Wilson coefficients C_i , $i = 1, \dots, 10$ and the LO $C_{7\gamma}^{eff}$ and C_{8g}^{eff} , at the scale $m_b = 4.2 \text{GeV}$ ($\sqrt{\Lambda_h m_b} = 1.45 \text{GeV}$) are given by [18]

$$\begin{aligned} C_1 &= 1.081(1.190) , \quad C_2 = -0.191(-0.373) \quad C_3 = 0.014(0.027) , \\ C_4 &= -0.036(-0.062) , \quad C_5 = 0.009(0.012) \quad C_6 = -0.042(-0.086) , \\ C_7/\alpha &= -0.001(-0.013) , \quad C_8/\alpha = 0.060(0.133) \quad C_9/\alpha = -1.254(-1.380) , \\ C_{10}/\alpha &= 0.223(0.432) , \quad C_{7\gamma}^{eff,LO} = -0.318 , \quad C_{8g}^{eff,LO} = -0.151 , \end{aligned} \quad (171)$$

where the parameters $\Lambda_{\overline{MS}}^{(5)} = 0.225 \text{GeV}$, $m_t(m_t) = 167 \text{GeV}$, $m_b(m_b) = 4.2 \text{GeV}$, $M_W = 80.4 \text{GeV}$, $\alpha = 1/129$, $\sin(\theta_W) = 0.23$, $\alpha_s(M_Z) = 0.118$ have been used. By using the input parameters and the Wilson coefficients, we list the values of a_i , $i = 1, \dots, 10$, and b_j ,

$j = 1, \dots, 3$, calculated at the scale $m_b = 4.2\text{GeV}$ as below

$$\begin{aligned}
a_1 &= 0.938 + 0.014i, & a_2 &= 0.351 - 0.081i, & a_3 &= -0.011 + 0.003i, \\
a_4^u &= -0.020 - 0.015i, & a_4^c &= -0.025 - 0.006i, & a_5 &= 0.017 - 0.003i, \\
r_\chi^K a_6^u &= -0.048 - 0.016i, & r_\chi^K a_6^c &= -0.054 - 0.006i, & a_7/\alpha &= 0.060 + 0.004i, \\
r_\chi^K a_8^u/\alpha &= 0.077 - 0.014i, & r_\chi^K a_8^c/\alpha &= 0.072 - 0.07i, & a_9/\alpha &= -1.149 - 0.017i, \\
a_{10}^u/\alpha &= -0.378 + 0.082i, & a_{10}^c/\alpha &= -0.382 + 0.088i, & r_A b_1 &= -0.154 - 0.022i, \\
r_A b_2 &= 0.048 + 0.007i, & r_A b_3 &= -0.003 - 0.0005i, & r_A b_4 &= 0.019 + 0.003i, \\
r_A b_3^{EW}/\alpha &= 0.179 + 0.026i, & r_A b_4^{EW}/\alpha &= -0.073 - 0.011i,
\end{aligned} \tag{172}$$

in which

$$r_A = \frac{B_{\pi K}}{A_{\pi K}} = \frac{f_B f_\pi}{m_B^2 F_0^{B \rightarrow \pi}(0)} \simeq 0.004, \tag{173}$$

and

$$r_\chi^K = \frac{2m_K^2}{\bar{m}_b(\bar{m}_q + \bar{m}_s)} \simeq 1.18(0.76), \tag{174}$$

where $r_\chi^K = 1.18$ is calculated at the scale $m_b = 4.2 \text{ GeV}$, and $r_\chi^K = 0.76$ is calculated at the scale $\sqrt{\Lambda_h m_b} = 1.45 \text{ GeV}$. Different contributions to the coefficients a_i are given in Table I for reference. The hard scattering contributions are dominant for a_3, a_5, a_7, a_{10} . The penguin contributions are minor for all $a_i, i = 1, \dots, 10$. The vertex contributions are important except of $a_{6,8}$.

The branching ratio for a $\bar{B} \rightarrow \pi K$ decay is given by the expression

$$Br(\bar{B} \rightarrow \pi K) = \frac{\tau_B}{16\pi m_B} |A(\bar{B} \rightarrow \pi K)|^2, \tag{175}$$

The predictions in the CE column of Table II are calculated according to the results derived in this paper. The predictions in BN column of Table II are quoted from the paper [20]. The predictions in DYZ column of Table II are quoted from the paper [21]. We observe that the predictions in the BN column are about 1.7 – 1.9 times larger than those in the CE and DYZ columns. To understand this difference, we employed the theoretic input parameters given in the Table 2 and the a_i and b_i values given in the Table 3-5 of [18] to recalculate the predictions for the branching ratios for the $B \rightarrow \pi K$ decays. The predictions are given in the BN' column of Table II. To check the consistency between our calculations made in

TABLE I: Different contributions to the coefficients a_i , $i = 1, \dots, 10$.

a_i	$C_i + \frac{C_{i\pm 1}}{N_c}$	$\frac{\alpha_s C_F C_{i\pm 1}}{4\pi N_c} V_i$	$\frac{\pi \alpha_s C_F C_{i\pm 1}}{N_c^2} H_i$	P_i^p	total
a_1	1.017	$0.028 + 0.014i$	-0.107	0	$0.938 + 0.014i$
a_2	0.169	$-0.158 - 0.081i$	0.340	0	$0.351 - 0.081i$
a_3	0.002	$0.005 + 0.003i$	-0.018	0	$-0.011 + 0.003i$
a_4^u	-0.029	$-0.002 - 0.001i$	0.008	$0.003 - 0.014i$	$-0.020 - 0.015i$
a_4^c	-0.029	$-0.002 - 0.001i$	0.008	$-0.002 - 0.005i$	$-0.025 - 0.006i$
a_5	-0.005	$-0.002 - 0.003i$	0.024	0	$0.017 - 0.003i$
a_6^u	-0.039	-4.27×10^{-4}	0	$-0.002 - 0.014i$	$-0.041 - 0.014i$
a_6^c	-0.039	-4.27×10^{-4}	0	$-0.007 - 0.005i$	$-0.046 - 0.005i$
a_7/α	0.019	$0.003 + 0.004i$	0.038	0	$0.060 + 0.004i$
a_8^u/α	0.060	-4.75×10^{-5}	0	$0.005 - 0.012i$	$0.065 - 0.012i$
a_8^c/α	0.060	-4.75×10^{-5}	0	$0.001 - 0.006i$	$0.061 - 0.006i$
a_9/α	-1.180	$-0.033 - 0.017i$	0.064	0	$-1.149 - 0.017i$
a_{10}^u/α	-0.195	$0.184 + 0.094i$	-0.395	$0.028 - 0.012i$	$-0.378 + 0.082i$
a_{10}^c/α	-0.195	$0.184 + 0.094i$	-0.395	$0.024 - 0.006i$	$-0.382 + 0.088i$

 TABLE II: Predictions for CP -averaged branching ratios (in unites of 10^{-6}) for $B \rightarrow \pi K$ decays.

mode	CE	BN'	BN[20]	DYZ[21]	data[74]
$B^- \rightarrow \pi^- \bar{K}^0$	10.1	12.1	19.3	12.6	23.1 ± 1.0
$B^- \rightarrow \pi^0 K^-$	6.4	5.3	11.1	6.7	12.8 ± 0.6
$\bar{B}^0 \rightarrow \pi^+ K^-$	9.6	10.2	16.7	9.1	19.4 ± 0.6
$\bar{B}^0 \rightarrow \pi^0 \bar{K}^0$	3.9	6.0	7.0	4.3	10.0 ± 0.6

the BN' column and those in [18], let's make an example for comparison. For example, the prediction for $Br(B^- \rightarrow \pi^- \bar{K}^0)$ decay is calculated to be 12.1×10^{-6} , which is close to the central value given in the Eq. (93) in the paper [18]

$$10^6 \times Br(B^- \rightarrow \pi^- \bar{K}^0) = [14.1_{-4.0}^{+6.4}(m_s)_{-3.6}^{+8.1}(X_A)] \times \left[\frac{F_0^{B \rightarrow \pi}(0)}{0.28} \right], \quad (176)$$

where the first error is due to the parameter variations shown in the Table 2 in that paper, and the second error is from the uncertainty due to power corrections from weak annihilation and twist-3 hard spectator contributions. We find that our calculation is consistent with that made in [18]. We argue that the difference between the predictions given in the BN' and BN columns maybe due to different methods used for determining the central values of the predictions among many sources of theoretical uncertainties. In average, the predictions made by the CE, BN' and DYZ schemes are only about one half of the experimental data. The predictions made by the BN scheme given in the paper [20] are consistent with but lower than the experimental data [74].

VI. DISCUSSIONS AND CONCLUSIONS

In this paper, we have shown that the factorizability of the QCDF amplitudes for $B \rightarrow PP$ decays with twist-3 two parton contributions can be preserved under the energetic meson limit that the two final state light mesons carry energetic momenta. Namely, we have extended the factorization theorem Eq. (1) to $O(\alpha_s)$ and $O(1/m_b)$ under the two parton approximation. The factorizability is shown by the following facts: (1) Under the energetic meson limit, the pseudoscalar distribution amplitude for a light pseudoscalar meson is allowed to be non-constant by the equations of motion for the quark. (2) The non-constant $\phi_p(x)$ is then used to regularize the end-point divergences in the hard spectator corrections at twist-3 order. (3) By retaining the dependence of the momentum fraction variable of the spectator quark of the B meson, the end-point divergent problem for the annihilation corrections at twist-3 order are solved. Based on the factorization for the matrix element at the twist-3 order, we have constructed a collinear expansion (CE) scheme for calculations of the hard scattering kernels of order $O(\alpha_s)$ and $O(\Lambda_{QCD}/m_b)$. The results were applied to make predictions for the branching ratios of $B \rightarrow \pi K$ decays. The predicted averaged branching ratios of $B \rightarrow \pi K$ decays are only about one half of the experimental data. Because the end point divergences in the hard spectator and annihilation corrections as found in previous studies have been regularized, the strong phase related to these two contributions is predicted to be universal as $\phi_A = 8.2^\circ$.

The predictions for the averaged branching ratios of $B \rightarrow \pi K$ decays made by three schemes, the CE, BN' and DYZ schemes, all have similar magnitudes. Although the CE

scheme contains no significant improvements in phenomenology than the other two schemes, however, it has reduced large uncertainties in the $X_{H,A}$ terms, theoretically. In literature, the $X_{H,A}$ terms are modeled as

$$X_{H,A} = \ln \frac{m_B}{\Lambda_h} (1 + \rho_{H,A} e^{i\phi_{H,A}}) , \quad (177)$$

where $\rho_{H,A} \leq 1$ and $\phi_{H,A}$ are unknown parameters and process dependent in general. In order to reduce the theoretical uncertainties, $\rho_{H,A}$ and $\phi_{H,A}$ are further assumed universal and their values are determined phenomenologically [20]. This reduces the prediction power of QCDF. On the other hand, in our approach, the relevant terms are finite and their values are calculable. The introduced B meson DA $\phi_B(\xi)$ in the weak annihilation and twist-3 hard spectator contributions results in no additional uncertainties than the other schemes, because the parameters in the $\phi_B(\xi)$ Eq. (161) are completely determined by the first two moments of the $\phi_B(\xi)$ Eq. (162). Since the most contributions of $A_{1,2}^i$ are from the end-point region, it is also interesting to check whether different models for the $\phi_B(\xi)$ can result in different values for $A_{1,2}^i$. As found in [75], the end-point behaviors of the model for the $\phi_B(\xi)$ are well controlled by the first two moments Eq. (162) and almost independent of the parameterization form of the assumed model. This implies that the $A_{1,2}^i$ given by Eq. (161) are almost model independent. In summary, the predictions under our approach are not only free from the end-point divergences but also independent of the model for $\phi_B(\xi)$. In addition, we have unproven the theoretical uncertainties to be of order $O(1/m_b^2)$.

We emphasize that the methods proposed in this work for resolving the end-point divergences is not ad hoc but general. For example, in the charmless B decays with one scalar meson in the final state, the chirally enhanced corrections are necessary. The twist-3 two parton DAs for the scalar meson can get involved. Similar to the pseudoscalar meson, the equation of motion for the twist-3 two parton DAs for the scalar meson are also used to determine the DAs [76, 77]. The physical situation in these decay processes is similar to that in $B \rightarrow \pi K$ decays. Therefore, the analysis given in Section II and the calculation scheme given in Section III can be used. We will show this fact in another place. Another important contribution of this paper is that the proposed CE scheme not only provides a systematic method for including higher twist contributions, but also is consistent with the QCD factorization scheme. This is the first method in the literature that can systematically calculate the higher twist contributions within the factorization approach. Last, the com-

plete twist-3 contributions need to consider the three particle DA. We plane to study this issue in another preparing work.

Acknowledgments

The author would like to appreciate the constant supports from the Department of Science Application and Dissemination and the National Science Council of R.O.C. under Grant Numbers NSC92-2112-M-142-001 , NSC93-2112-M-142-001 and NSC95-2112-M-142-001.

APPENDIX A: CALCULATIONS OF TWIST-3 VERTEX CORRECTIONS

In this Appendix, the calculations for the one loop vertex contributions to the matrix element of operators Q_i , $i = 6$ will be present. The amplitude for the Feynman diagram as depicted in Fig. 3(a) is written as

$$\begin{aligned}
& \langle Q_5^{(8)(V-A)(V+A)} \rangle_{(a)}^{vertex,1-loop} \\
&= if_{M_2} \mu_{M_2} \frac{\pi \alpha_s C_F \mu^{2D}}{N_c} \int_0^1 du \Gamma(3) \int_0^1 dx \int_0^{1-x} dy \int \frac{d^D k}{(2\pi)^D} \frac{1}{[k^2 - 2(xP_b + yuq) \cdot k]^3} \\
& \quad \times \langle M_1 | \bar{q} \gamma^\mu (1 + \gamma_5) (\gamma_5 \hat{\phi}_p^{M_2}(u) - \frac{1}{2} \epsilon_\perp \cdot \sigma \hat{\phi}_\sigma^{M_2}(u)) \gamma^\alpha (u \not{d} - \not{k}) \gamma_\mu (1 - \gamma_5) (\not{P}_b - \not{k} + m_b) \gamma_\alpha b | \bar{B} \rangle .
\end{aligned} \tag{A1}$$

where NDR has been used. In the above expression, we have used the following spin state expansion for the matrix element $\langle M_2 | \bar{q}(0) q(\lambda n/E) | 0 \rangle$

$$\int_0^{infy} \frac{d\lambda}{2\pi} e^{-i\bar{u}\lambda} \langle M_2 | \bar{q}(0) q(\lambda n/E) | 0 \rangle = -\frac{if_{M_2}}{4N_c} [\gamma_5 \not{d} \phi_P^{M_2}(u) + \mu_\chi^{M_2} \left(\gamma_5 \hat{\phi}_p^{M_2}(u) - \frac{1}{2} \epsilon_\perp \cdot \sigma \hat{\phi}_\sigma^{M_2}(u) \right)] . \tag{A2}$$

Perform a substitution of $k \rightarrow k + xP_b + yuq$ for k to arrive at

$$\begin{aligned}
& \langle Q_5^{(V-A)(V+A)} \rangle_{(a)}^{vertex,1-loop} \\
&= if_{M_2} \mu_{M_2} \frac{\pi \alpha_s C_F \mu^{2D}}{N_c} \int_0^1 du \hat{\phi}_p^{M_2}(u) \Gamma(3) \int_0^1 dx \int_0^{1-x} dy \int \frac{d^D k}{(2\pi)^D} \frac{1}{[k^2 - (xP_b + yuq)^2]^3} \\
& \quad \times \langle M_1 | \bar{q} \gamma^\mu (1 + \gamma_5) (\gamma_5 - \frac{1}{2} \epsilon_\perp \cdot \sigma) \gamma^\alpha (u \bar{y} \not{d} - x \not{P}_b - \not{k}) \gamma_\mu (1 - \gamma_5) (\bar{x} \not{P}_b - yu \not{d} - \not{k} + m_b) \gamma_\alpha b | \bar{B} \rangle ,
\end{aligned} \tag{A3}$$

where we have used $\hat{\phi}_p^{M_2}(u) = \hat{\phi}_\sigma^{M_2}(u)$. The contributions with odd number of k give vanishing results. Completing the loop integration over k by NDR gives

$$\begin{aligned}
& \langle Q_5^{(8)(V-A)(V+A)} \rangle_{(a)}^{vertex,1-loop} \\
&= if_{M_2\mu M_2} \frac{\pi\alpha_s C_F \mu^{2D}}{N_c} \int_0^1 du \hat{\phi}_p^{M_2}(u) \Gamma(3) \int_0^1 dx \int_0^{1-x} dy \frac{(-1)^3}{(4\pi)^{D/2}} \frac{\Gamma(3-D/2)}{\Gamma(3)} \left(\frac{1}{(xP_b + yuq)^2} \right)^{3-D/2} \\
&\quad \times \left\{ \langle M_1 | \bar{q} \gamma^\mu (1 + \gamma_5) (\gamma_5 - \frac{1}{2} \epsilon_\perp \cdot \sigma) \gamma^\alpha (u \bar{y} \not{q} - x \not{P}_b) \gamma_\mu (1 - \gamma_5) (\bar{x} \not{P}_b - yu \not{q} + m_b) \gamma_\alpha b | \bar{B} \rangle \right. \\
&\quad \left. - \frac{g^{\eta\lambda}}{2} \frac{\Gamma(2-D/2)}{\Gamma(3-D/2)} \left(\frac{1}{(xP_b + yuq)^2} \right)^{-1} \langle M_1 | \bar{q} \gamma^\mu (1 + \gamma_5) (\gamma_5 - \frac{1}{2} \epsilon_\perp \cdot \sigma) \gamma^\alpha \gamma_\eta \gamma_\mu (1 - \gamma_5) \gamma_\lambda \gamma_\alpha b | \bar{B} \rangle \right\} .
\end{aligned} \tag{A4}$$

We note that the first term in the bracket of Eq. (A4) is finite under $x, y \rightarrow 1$ while becomes divergent as $x, y \rightarrow 0$, and the second term in the bracket of Eq. (A4) is finite under $x, y \rightarrow 0$ while becomes divergent as $x, y \rightarrow 1$. This implies that the first term is infrared divergent while the second term is ultra-violate divergent. We apply $D = 4 + 2a$ for the first term and $D = 4 - 2\epsilon$ for the second term.

The Dirac identities in D -dimension

$$\begin{aligned}
\gamma^\mu \gamma_\mu &= D , \\
\gamma^\mu \gamma^\nu \gamma_\mu &= -(D-2) \gamma^\nu , \\
\gamma^\mu \gamma^\alpha \gamma^\beta \gamma_\mu &= 4g^{\alpha\beta} - (4-D) \gamma^{\alpha\beta} , \\
\gamma^\mu \gamma^\alpha \gamma^\beta \gamma^\gamma \gamma_\mu &= -2\gamma^\gamma \gamma^\alpha \gamma^\beta + (4-D) \gamma^\alpha \gamma^\beta \gamma^\gamma ,
\end{aligned} \tag{A5}$$

can be used to simplify the Dirac matrix. After completing the spin algebra, we found that only $\gamma_5 \hat{\phi}_p^{M_2}(u)$ can contribute. The UV part of $\langle Q_5^{(8)(V-A)(V+A)} \rangle_{(a)}^{vertex,1-loop}$ is

$$\begin{aligned}
& \langle Q_5^{(8)(V-A)(V+A)} \rangle_{(a),UV}^{vertex,1-loop} \\
&= -2if_{M_2\mu M_2} \langle M_1 | \bar{q} (1 - \gamma_5) b | \bar{B} \rangle \frac{\alpha_s C_F}{4\pi N_c} \int_0^1 du \hat{\phi}_p^{M_2}(u) \\
&\quad \times (1 - \frac{5}{2}\epsilon) \left(\frac{4\pi\mu^2}{m_b^2} \right)^\epsilon \Gamma(\epsilon) \int_0^1 dx \int_0^{1-x} dy \frac{2}{[x(x+yu)]^\epsilon} .
\end{aligned} \tag{A6}$$

The integrals over x and y are

$$\int_0^1 dx \int_0^{1-x} dy \frac{2}{[x(x+yu)]^\epsilon} = 1 + 3\epsilon + \frac{u \ln u}{\bar{u}} . \tag{A7}$$

By substituting the above identity, the UV contribution becomes

$$\begin{aligned}
& \langle Q_5^{(8)(V-A)(V+A)} \rangle_{(a),UV}^{vertex,1-loop} \\
&= -2if_{M_2}\mu_{M_2} \langle M_1 | \bar{q}(1 - \gamma_5)b | \bar{B} \rangle \frac{\alpha_s C_F}{4\pi N_c} \int_0^1 du \hat{\phi}_p^{M_2}(u) \\
&\quad \times \left(\frac{1}{\epsilon} - \gamma_E + 4\pi + \ln \frac{\mu^2}{m_b^2} + \frac{1}{2} + \frac{u \ln u}{\bar{u}} \right). \tag{A8}
\end{aligned}$$

The calculation for the IR part of $\langle Q_5^{(8)(V-A)(V+A)} \rangle_{(a)}^{vertex,1-loop}$ requires some cares. As we have shown in Sec.III, the different components of the vertexes γ^μ , γ^α , γ_α lead to contributions of different magnitudes. The IR part of $\langle Q_5^{(V-A)(V+A)} \rangle_{(a)}^{vertex,1-loop}$ appears as

$$\begin{aligned}
& \langle Q_5^{(8)(V-A)(V+A)} \rangle_{(a)}^{vertex,1-loop} \\
&= if_{M_2}\mu_{M_2} \frac{\pi\alpha_s C_F \mu^{2D}}{N_c} \int_0^1 du \hat{\phi}_p^{M_2}(u) \Gamma(3) \int_0^1 dx \int_0^{1-x} dy \int \frac{d^D k}{(2\pi)^D} \frac{1}{[k^2 - 2(xP_b + yuq) \cdot k]^3} \\
&\quad \times \langle M_1 | \bar{q} \gamma^\mu (1 + \gamma_5) \gamma_5 (2(\bar{y}u - x)q^\alpha - x\gamma^\alpha \not{p}) \gamma_\mu (1 - \gamma_5) (2\bar{x}P_b^\alpha + (xm_b - yu\not{q})\gamma_\alpha) b | \bar{B} \rangle. \tag{A9}
\end{aligned}$$

By some manipulations, the expression becomes

$$\begin{aligned}
& \langle Q_5^{(V-A)(V+A)} \rangle_{(a),IR}^{vertex,1-loop} \\
&= 2if_{M_2}\mu_{M_2} \langle M_1 | \bar{q}(1 - \gamma_5)b | \bar{B} \rangle \frac{\alpha_s C_F}{16\pi N_c} \int_0^1 du \hat{\phi}_p^{M_2}(u) \\
&\quad \times \left(\frac{4\pi\mu^2}{m_b^2} \right)^{-a} \Gamma(1-a) \int_0^1 dx \int_0^{1-x} dy \frac{N_{IR}}{[x(x+yu)]^{1-a}}, \tag{A10}
\end{aligned}$$

where

$$N_{IR} = [2Du\bar{x}\bar{y} + 4uxy - (4 + 2D)x\bar{x}].$$

After completing the integrations over x and y , the $\langle Q_5^{(8)(V-A)(V+A)} \rangle_{(a),IR}^{vertex,1-loop}$ becomes

$$\begin{aligned}
& \langle Q_5^{(V-A)(V+A)} \rangle_{(a),IR}^{vertex,1-loop} \\
&= 2if_{M_2}\mu_{M_2} \langle M_1 | \bar{q}(1 - \gamma_5)b | \bar{B} \rangle \frac{\alpha_s C_F}{4\pi N_c} \left(\frac{4\pi\mu^2}{m_b^2} \right)^{-a} \Gamma(1-a) \int_0^1 du \hat{\phi}_p^{M_2}(u) \\
&\quad \times \left[\frac{1}{a^2} + \frac{2 \ln u}{a} + \frac{1}{2a} - \ln u + \ln^2 u - 2Li_2(1 - \frac{1}{u}) + 1 + \frac{3 \ln u}{\bar{u}} \right]. \tag{A11}
\end{aligned}$$

As a result, $\langle Q_5^{(V-A)(V+A)} \rangle_{(a)}^{vertex,1-loop}$ is written as

$$\langle Q_5^{(V-A)(V+A)} \rangle_{(a)}^{vertex,1-loop}$$

$$\begin{aligned}
&= -2if_{M_2}\mu_{M_2}\langle M_1|\bar{q}(1-\gamma_5)b|\bar{B}\rangle\frac{\alpha_s C_F}{4\pi N_c}\int_0^1 du\hat{\phi}_p^{M_2}(u)\left\{\left(\frac{1}{\epsilon}-\gamma_E+4\pi+\ln\frac{\mu^2}{m_b^2}+\frac{1}{2}+\frac{u\ln u}{\bar{u}}\right)\right. \\
&\quad \left.-\left(\frac{4\pi\mu^2}{m_b^2}\right)^{-a}\Gamma(1-a)\left[\frac{1}{a^2}+\frac{2\ln u}{a}+\frac{1}{2a}-\ln u+\ln^2 u-2Li_2\left(1-\frac{1}{u}\right)+1+\frac{3\ln u}{\bar{u}}\right]\right\}.
\end{aligned} \tag{A12}$$

The calculations for the Feynman diagrams depicted in Fig. 3(b-d) can be done in a similar way. The amplitudes for these three diagrams are written as

$$\begin{aligned}
&\langle Q_5^{(V-A)(V+A)}\rangle_{(b)}^{vertex,1-loop} \\
&= 2if_{M_2}\mu_{M_2}\langle M_1|\bar{q}(1-\gamma_5)b|\bar{B}\rangle\frac{\alpha_s C_F}{4\pi N_c}\int_0^1 du\hat{\phi}_p^{M_2}(u)\left\{\left(\frac{1}{\epsilon}-\gamma_E+4\pi+\ln\frac{\mu^2}{m_b^2}+\frac{7}{2}+\frac{\bar{u}\ln\bar{u}}{u}\right)\right. \\
&\quad \left.-\left(\frac{4\pi\mu^2}{m_b^2}\right)^{-a}\Gamma(1-a)\left[\frac{1}{a^2}+\frac{2\ln\bar{u}}{a}+\frac{1}{2a}-\ln\bar{u}+\ln^2\bar{u}-2Li_2\left(1-\frac{1}{\bar{u}}\right)+1+\frac{3\ln\bar{u}}{u}\right]\right\}.
\end{aligned} \tag{A13}$$

$$\begin{aligned}
&\langle Q_5^{(V-A)(V+A)}\rangle_{(c)}^{vertex,1-loop} \\
&= 2if_{M_2}\mu_{M_2}\langle M_1|\bar{q}(1-\gamma_5)b|\bar{B}\rangle\frac{\alpha_s C_F}{4\pi N_c}\int_0^1 du\hat{\phi}_p^{M_2}(u)\left\{\left(\frac{1}{\epsilon}-\gamma_E+4\pi+\ln\frac{\mu^2}{m_b^2}+\frac{7}{2}-\ln u+i\pi\right)\right. \\
&\quad \left.-\left(\frac{4\pi\mu^2}{m_b^2}\right)^{-a}\Gamma(1-a)\left[\frac{2}{a^2}+\frac{2\ln u}{a}-\frac{3+2i\pi}{2a}-(4+2i\pi)\ln u+4i\pi+\ln^2 u+\frac{27+2\pi}{6}\right]\right\}.
\end{aligned} \tag{A14}$$

$$\begin{aligned}
&\langle Q_5^{(V-A)(V+A)}\rangle_{(d)}^{vertex,1-loop} \\
&= -2if_{M_2}\mu_{M_2}\langle M_1|\bar{q}(1-\gamma_5)b|\bar{B}\rangle\frac{\alpha_s C_F}{4\pi N_c}\int_0^1 du\hat{\phi}_p^{M_2}(u)\left\{\left(\frac{1}{\epsilon}-\gamma_E+4\pi+\ln\frac{\mu^2}{m_b^2}+\frac{1}{2}-\ln\bar{u}+i\pi\right)\right. \\
&\quad \left.-\left(\frac{4\pi\mu^2}{m_b^2}\right)^{-a}\Gamma(1-a)\left[\frac{2}{a^2}+\frac{2\ln\bar{u}}{a}-\frac{3+2i\pi}{2a}-(4+2i\pi)\ln\bar{u}+4i\pi+\ln^2\bar{u}+\frac{27+2\pi}{6}\right]\right\}.
\end{aligned} \tag{A15}$$

The summation of the above contributions from Fig. 3 (a)-(d) gives the $V_{6,8}$ in Eq. (88). It is obvious that the IR divergences are cancelled. The UV divergences are regularized by means of the dimensional regularization and the remaining scheme constants are subtracted by the \overline{MS} subtraction scheme. Our calculation is consistent with the result derived by using the BN scheme [20]. We also note that, for the vertex contributions, the pseudotensor LCDA does not contribute in both CE and BN schemes. In the DYZ scheme, the pseudotensor LCDA can involve in the vertex contributions and there are associated IR divergences.

The authors in [23] argued that a symmetric parameterization for pseudotensor LCDA is necessary for eliminating the associated IR divergences. Although both CE and BN schemes obtain similar results for the vertex contribution $V_6(M_2)$, the meanings for the pseudoscalar LCDA ($\hat{\phi}_p^{M_2}(u)$ in the CE scheme and the $\phi_p^{M_2}(u)$ in the BN scheme) are different. In the CE scheme, the $\hat{\phi}_p^{M_2}(u) = 6u\bar{u}$ is used. On the other hand, in the BN scheme, $\phi_p^{M_2}(u) = 1$ was used.

APPENDIX B: CALCULATIONS OF TWIST-3 PENGUIN CORRECTIONS

The calculations for twist-3 penguin contributions from the penguin contractions as depicted in the diagrams in Fig. 3 (e) and (f) are given in this Appendix. We denote the penguin topology in Fig. 3 (e) as type-*I* and penguin topology in Fig. 3 (f) as type-*II*. The contributions from $Q_{1,3}$ appear in the type-*I* penguin topology and the contributions from $Q_{4,6}$ are identified as the type-*II* penguin topology. For the type-*I* penguin topology, the amplitude from operators $Q_{1,3}$ are written as

$$\begin{aligned} & \langle Q_{1,3}^{(V-A)(V-A)} \rangle_{penguin,1-loop} \\ &= \frac{C_F \pi \alpha_s \mu^{4-D}}{N_c} f_{M_2} \mu_\chi^{M_2} \int_0^1 du \int_0^1 dx \int \frac{d^D k}{(2\pi)^D} \frac{1}{[k^2 - R^2]^2} \\ & \times \frac{1}{(p-uq)^2} \langle M_1 | \bar{q} \gamma^\alpha (\gamma_5 \hat{\phi}_p^{M_2}(u) - \frac{1}{2} \epsilon_\perp \cdot \sigma \hat{\phi}_\sigma^{M_2}(u)) \gamma^\mu (1 - \gamma_5) \\ & (\not{k} - \bar{x}(\not{p} - u\not{q}) + m_q) \gamma_\alpha (\not{k} + x(\not{p} - u\not{q}) + m_q) \gamma_\mu (1 - \gamma_5) b | B \rangle, \end{aligned} \quad (B1)$$

where $R^2 = m_b^2(s_q - x\bar{x}u)$ with $s_q = m_q^2/m_b^2$. For the type-*II* penguin topology, the relevant amplitude from operators $Q_{4,6}$ is expressed as

$$\begin{aligned} & \langle Q_{4,6}^{(V-A)(V-A)} \rangle_{penguin,1-loop} \\ &= -\frac{C_F \pi \alpha_s \mu^{4-D}}{N_c} f_{M_2} \mu_\chi^{M_2} \int_0^1 du \int_0^1 dx \int \frac{d^D k}{(2\pi)^D} \frac{1}{[k^2 - R^2]^2} \\ & \times \frac{1}{(p-uq)^2} \langle M_1 | \bar{q} \gamma^\alpha (\gamma_5 \hat{\phi}_p^{M_2}(u) - \frac{1}{2} \epsilon_\perp \cdot \sigma \hat{\phi}_\sigma^{M_2}(u)) \gamma^\mu (1 - \gamma_5) b | B \rangle \\ & \times \sum_{q=u,c,b} \{ \text{Tr}[\gamma_\alpha (\not{k} - \bar{x}(\not{p} - u\not{q}) + m_q) \gamma_\mu (1 - \gamma_5) (\not{k} + x(\not{p} - u\not{q}) + m_q)] \}. \end{aligned} \quad (B2)$$

where $R^2 = m_b^2(s_q - x\bar{x}u)$ with $s_q = m_q^2/m_b^2$. We first complete the k integration to obtain

$$\langle Q_{1,3}^{(V-A)(V-A)} \rangle_{penguin,1-loop}$$

$$\begin{aligned}
&= -if_{M_2}\mu_\chi^{M_2} \frac{C_F\alpha_s}{16\pi N_c} \int_0^1 du \int_0^1 dx \\
&\quad \times \left\{ \frac{g_{\eta\lambda}}{2} R^2 \langle M_1 | \bar{q} \gamma^\alpha (\gamma_5 \hat{\phi}_p^{M_2}(u) - \frac{1}{2} \epsilon_\perp \cdot \sigma \hat{\phi}_\sigma^{M_2}(u)) \gamma^\mu (1 - \gamma_5) \gamma^\lambda \gamma_\alpha \gamma^\eta \gamma_\mu (1 - \gamma_5) b | B \rangle \right. \\
&\quad \left. + x\bar{x} \left(\frac{4\pi\mu^2}{m_b^2} \right)^\epsilon \frac{\Gamma(\epsilon)}{\bar{R}^{2\epsilon}} \langle M_1 | \bar{q} \gamma^\alpha (\gamma_5 \hat{\phi}_p^{M_2}(u) - \frac{1}{2} \epsilon_\perp \cdot \sigma \hat{\phi}_\sigma^{M_2}(u)) \gamma^\mu (1 - \gamma_5) ((\not{p} - u\not{q}) + m_q) \right. \\
&\quad \left. \times \gamma_\alpha ((\not{p} - u\not{q}) + m_q) \gamma_\mu (1 - \gamma_5) b | B \rangle \right\} \frac{1}{(p - uq)^2} \quad (B3)
\end{aligned}$$

and

$$\begin{aligned}
&\langle Q_{4,6}^{(V-A)(V\pm A)} \rangle_{penguin,1-loop} \\
&= if_{M_2}\mu_\chi^{M_2} \frac{C_F\alpha_s}{16\pi N_c} \int_0^1 du \int_0^1 dx \\
&\quad \times \left\{ \frac{g_{\eta\lambda}}{2} R^2 \langle M_1 | \bar{q} \gamma^\alpha (\gamma_5 \hat{\phi}_p^{M_2}(u) - \frac{1}{2} \epsilon_\perp \cdot \sigma \hat{\phi}_\sigma^{M_2}(u)) \gamma^\mu (1 - \gamma_5) b | B \rangle \text{Tr}[\gamma_\alpha \gamma^\lambda \gamma_\mu (1 - \gamma_5) \gamma^\eta] \right. \\
&\quad \left. + x\bar{x} \left(\frac{4\pi\mu^2}{m_b^2} \right)^\epsilon \frac{\Gamma(\epsilon)}{\bar{R}^{2\epsilon}} \langle M_1 | \bar{q} \gamma^\alpha (\gamma_5 \hat{\phi}_p^{M_2}(u) - \frac{1}{2} \epsilon_\perp \cdot \sigma \hat{\phi}_\sigma^{M_2}(u)) \gamma^\mu (1 - \gamma_5) b | B \rangle \right. \\
&\quad \left. \times \left(\sum_{q=u,c,b} \text{Tr}[\gamma_\alpha ((\not{p} - u\not{q}) + m_q) \gamma_\mu (1 - \gamma_5) ((\not{p} - u\not{q}) + m_q)] \right) \right\} \frac{1}{(p - uq)^2}. \quad (B4)
\end{aligned}$$

where $\bar{R} = \sqrt{s_q - x\bar{x}u}$. After completing the spin algebra, we arrive at

$$\begin{aligned}
\langle Q_{1,3}^{(V-A)(V-A)} \rangle_{penguin,1-loop} &= 4if_{M_2}\mu_\chi^{M_2} \langle M_1 | \bar{q} (1 - \gamma_5) b | B \rangle \frac{C_F\alpha_s}{4\pi N_c} \int_0^1 du \hat{\phi}_p^{M_2}(u) \int_0^1 dx x\bar{x} \\
&\quad \left[\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) + \ln \frac{\mu^2}{m_b^2} - 2 - 2 \ln \sqrt{s_q - x\bar{x}u} \right] \quad (B5)
\end{aligned}$$

and

$$\begin{aligned}
\langle Q_{4,6}^{(V-A)(V-A)} \rangle_{penguin,1-loop} &= 4if_{M_2}\mu_\chi^{M_2} \langle M_1 | \bar{q} (1 - \gamma_5) b | B \rangle \frac{C_F\alpha_s}{4\pi N_c} \int_0^1 du \hat{\phi}_p^{M_2}(u) \int_0^1 dx x\bar{x} \\
&\quad \sum_{q=u,c,b} \left[\frac{1}{\epsilon} - \gamma_E + \ln(4\pi) + \ln \frac{\mu^2}{m_b^2} - 2 \ln \sqrt{s_q - x\bar{x}u} \right]. \quad (B6)
\end{aligned}$$

After completing the integral over x , we obtain the contributions from different operators contained in the $P_6(M_2)$ function. Similar to the vertex contributions, the pseudotensor LCDA does not get involved in the penguin contributions. The penguin contributions calculated by the CE scheme are consistent with the results calculated by using the BN scheme [20], while are different from the results calculated by the DYZ scheme [23].

The calculation for the contributions from the chromo-magnetic dipole operator Q_{8G} is straightforward in the CE scheme. The amplitude is written as

$$-i \frac{f_{M_2}\mu_\chi^{M_2}\alpha_s m_b}{8\pi N_c} \int_0^1 du \langle M_1 | \bar{q} \gamma^\alpha (\gamma_5 \hat{\phi}_p^{M_2}(u) - \frac{1}{2} \epsilon \cdot \sigma \hat{\phi}_\sigma^{M_2}(u)) [\not{k} \gamma_\alpha - \gamma_\alpha \not{k}] (1 + \gamma_5) b | B \rangle \frac{1}{(P_b - uq)^2}. \quad (B7)$$

After completing the spin algebra, we arrive at

$$\begin{aligned}
& -i \frac{f_{M_2} \mu_{M_2} \alpha_s}{2\pi N_c} \int_0^1 du \langle M_1 | \bar{q} \left(\frac{3}{2} \hat{\phi}_p^{M_2}(u) + \frac{1}{2} \hat{\phi}_\sigma^{M_2}(u) \right) (1 - \gamma_5) b | B \rangle \\
& = (-4i f_{M_2} \mu_{M_2}) \frac{\alpha_s C_F}{4\pi N_c} \int_0^1 du \hat{\phi}_p^{M_2}(u) \langle M_1 | \bar{q} (1 - \gamma_5) b | B \rangle,
\end{aligned} \tag{B8}$$

where the second line is obtained by using the equations of motion Eq. (5). For comparison, we note that the calculation for the contributions from the chromo-magnetic dipole operator Q_{8G} by the BN scheme gives the following expression [20]

$$\begin{aligned}
& -i \frac{f_{M_2} \mu_{M_2} \alpha_s}{2\pi N_c} \int_0^1 du \langle M_1 | \bar{q} \left(\frac{3}{2} \phi_p(u) + \frac{1}{2} \frac{\phi'_\sigma(u)}{6} + \frac{1}{\bar{u}} \frac{\phi_\sigma(u)}{6} \right) (1 - \gamma_5) b | B \rangle \\
& = (-4i f_{M_2} \mu_{M_2}) \frac{\alpha_s C_F}{4\pi N_c} \int_0^1 du \phi_p(u) \langle M_1 | \bar{q} (1 - \gamma_5) b | B \rangle,
\end{aligned} \tag{B9}$$

where the second line is derived by using the Eq. (4). We note that the results from the penguin contractions calculated by the DYZ scheme are quite different from our results. The detailed expressions for the penguin contributions by the DYZ scheme are referred to their original paper [23].

APPENDIX C: WILSON COEFFICIENTS

The NLO Wilson coefficients used in this paper are summarized below for reference. The solution to the renormalization group equation for the Wilson coefficients C_1, \dots, C_{10} can be written as

$$\vec{C}(\mu) = \left[\mathbf{U}_0 + \frac{\alpha_s(\mu)}{4\pi} \mathbf{J}_0 \mathbf{U}_0 - \frac{\alpha_s(M_W)}{4\pi} \mathbf{U}_0 \mathbf{J}_0 + \frac{\alpha}{4\pi} \left(\frac{4\pi}{\alpha_s(\mu)} \mathbf{R}_0 + \mathbf{R}_1 \right) \right] \vec{C}(M_W) \tag{C1}$$

where the U_0 and R_0 are the LO parts and J_0 and R_1 are the NLO parts of C_i , respectively. The α_s is used its NLO expression. The expressions for the U_0, J_0, R_0 and R_1 matrices can be found in [73].

-
- [1] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **89** (2002) 201802 hep-ex/0207042.
 - [2] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **94**, 161803 (2005) hep-ex/0408127.
 - [3] K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. **87**, 091802 (2001) hep-ex/0107061.
 - [4] K. F. Chen *et al.* [Belle Collaboration], Phys. Rev. Lett. **98**, 031802 (2007) hep-ex/0608039.

- [5] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **89**, 281802 (2002) hep-ex/0207055.
- [6] Y. Chao *et al.* [Belle Collaboration], Phys. Rev. D **69**, 111102 (2004) hep-ex/0311061.
- [7] A. Bornheim *et al.* [CLEO Collaboration], Phys. Rev. D **68**, 052002 (2003) [Erratum-ibid. D **75**, 119907 (2007)] hep-ex/0302026.
- [8] K. Abe *et al.*, hep-ex/0507045.
- [9] Y. Chao *et al.* [Belle Collaboration], Phys. Rev. Lett. **93**, 191802 (2004) hep-ex/0408100.
- [10] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. Lett. **93**, 131801 (2004) hep-ex/0407057.
- [11] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **94**, 181802 (2005) hep-ex/0412037.
- [12] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **91**, 241801 (2003) hep-ex/0308012.
- [13] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **91**, 021801 (2003) hep-ex/0303028.
- [14] J. Charles *et al.* [CKMfitter Group], Eur. Phys. J. C **41**, 1 (2005) hep-ph/0406184.
- [15] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999) hep-ph/9905312.
- [16] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. Lett. **87** (2001) 201806 hep-ph/0107002.
- [17] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B **591**, 313 (2000) hep-ph/0006124.
- [18] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B **606**, 245 (2001) hep-ph/0104110.
- [19] C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D **70** (2004) 054015 hep-ph/0401188.
- [20] M. Beneke and M. Neubert, Nucl. Phys. B **675**, 333 (2003) hep-ph/0308039.
- [21] D. s. Du, H. u. Gong, J. f. Sun, D. s. Yang and G. h. Zhu, Phys. Rev. D **65**, 074001 (2002) hep-ph/0108141.
- [22] D. S. Du, C. S. Huang, Z. T. Wei and M. Z. Yang, Phys. Lett. B **520**, 50 (2001) [Erratum-ibid. B **530**, 258 (2002)] hep-ph/0107320.
- [23] D. s. Du, D. s. Yang and G. h. Zhu, Phys. Rev. D **64**, 014036 (2001) hep-ph/0103211.
- [24] D. s. Du, D. Yang and G. Zhu, Phys. Lett. B **509**, 263 (2001) hep-ph/0102077.
- [25] H. n. Li, S. Mishima and A. I. Sanda, Phys. Rev. D **72**, 114005 (2005) hep-ph/0508041.
- [26] C. W. Bauer, S. Fleming and M. E. Luke, Phys. Rev. D **63**, 014006 (2001) hep-ph/0005275.
- [27] C. W. Bauer, S. Fleming, D. Pirjol and I. W. Stewart, Phys. Rev. D **63**, 114020 (2001) hep-ph/0011336.

- [28] C. W. Bauer, D. Pirjol and I. W. Stewart, Phys. Rev. D **65** (2002) 054022 hep-ph/0109045.
- [29] M. Beneke, A. P. Chapovsky, M. Diehl and T. Feldmann, Nucl. Phys. B **643**, 431 (2002) hep-ph/0206152.
- [30] M. Beneke and T. Feldmann, Phys. Lett. B **553**, 267 (2003) hep-ph/0211358.
- [31] J. Chay and C. Kim, Phys. Rev. D **65**, 114016 (2002) hep-ph/0201197.
- [32] R. J. Hill and M. Neubert, Nucl. Phys. B **657**, 229 (2003) hep-ph/0211018.
- [33] T. Becher, R. J. Hill and M. Neubert, Phys. Rev. D **69**, 054017 (2004) hep-ph/0308122.
- [34] H. n. Li and H. L. Yu, Phys. Rev. Lett. **74**, 4388 (1995) hep-ph/9409313.
- [35] H. N. Li and H. L. Yu, Phys. Lett. B **353**, 301 (1995).
- [36] H. n. Li and H. L. Yu, Phys. Rev. D **53**, 2480 (1996) hep-ph/9411308.
- [37] Y. Y. Keum, H. n. Li and A. I. Sanda, Phys. Lett. B **504**, 6 (2001) hep-ph/0004004.
- [38] Y. Y. Keum, H. N. Li and A. I. Sanda, Phys. Rev. D **63**, 054008 (2001) hep-ph/0004173.
- [39] Y. Y. Keum and H. n. Li, Phys. Rev. D **63**, 074006 (2001) hep-ph/0006001.
- [40] C. D. Lu, K. Ukai and M. Z. Yang, Phys. Rev. D **63**, 074009 (2001) hep-ph/0004213.
- [41] T. W. Yeh and H. n. Li, Phys. Rev. D **56**, 1615 (1997) hep-ph/9701233.
- [42] C. H. Chang and H. n. Li, Phys. Rev. D **55**, 5577 (1997) hep-ph/9607214.
- [43] J. C. Collins, D. E. Soper and G. Sterman, Nucl. Phys. B **308**, 833 (1988).
- [44] M. Beneke and T. Feldmann, Nucl. Phys. B **592**, 3 (2001) hep-ph/0008255.
- [45] X. q. Li and Y. d. Yang, Phys. Rev. D **72**, 074007 (2005) hep-ph/0508079.
- [46] M. Beneke and D. Yang, Nucl. Phys. B **736**, 34 (2006) hep-ph/0508250.
- [47] M. Beneke and S. Jager, PoS **HEP2005**, 259 (2006) hep-ph/0512101.
- [48] M. Beneke and S. Jager, Nucl. Phys. B **751**, 160 (2006) hep-ph/0512351.
- [49] N. Kivel, JHEP **0705**, 019 (2007) hep-ph/0608291.
- [50] G. Bell, arXiv:0705.3133[hep-ph].
- [51] G. Bell, arXiv:0705.3127[hep-ph].
- [52] A. Jain, I. Z. Rothstein and I. W. Stewart, arXiv:0706.3399[hep-ph].
- [53] V. Pilipp, arXiv:0709.0497[hep-ph].
- [54] R. K. Ellis, W. Furmanski and R. Petronzio, Nucl. Phys. B **207**, 1 (1982).
- [55] R. K. Ellis, W. Furmanski and R. Petronzio, Nucl. Phys. B **212**, 29 (1983).
- [56] J. W. Qiu, Phys. Rev. D **42**, 30 (1990).
- [57] T. W. Yeh, accepted for publication in Chin. Journ. of Phys. arXiv:0802.1855 [hep-ph].

- [58] H. D. Politzer, Nucl. Phys. B **172**, 349 (1980).
- [59] T. W. Yeh, hep-ph/0107018.
- [60] T. W. Yeh, *Prepared for 4th International Conference on B Physics and CP Violation (BCP 4), Ago Town, Mie Prefecture, Japan, 19-23 Feb 2001.*
- [61] T. W. Yeh, Phys. Rev. D **66**, 014002 (2002).
- [62] T. W. Yeh, Phys. Rev. D **65**, 074016 (2002) hep-ph/0107192.
- [63] T. W. Yeh, Phys. Rev. D **65**, 094019 (2002). hep-ph/0204264.
- [64] V. M. Braun and I. E. Filyanov, Sov. J. Nucl. Phys. **52**, 126 (1990) [Z. Phys. C **48**, 239 (1990 YAFIA,52,199-213.1990)].
- [65] P. Ball, V. M. Braun, Y. Koike and K. Tanaka, Nucl. Phys. B **529**, 323 (1998) hep-ph/9802299.
- [66] P. Ball, JHEP **9901**, 010 (1999). hep-ph/9812375.
- [67] C. M. Arnesen, Z. Ligeti, I. Z. Rothstein and I. W. Stewart, hep-ph/0607001.
- [68] C. M. Arnesen, I. Z. Rothstein and I. W. Stewart, Phys. Lett. B **647** (2007) 405 [Erratum-ibid. B **653** (2007) 450] hep-ph/0611356.
- [69] M. Bauer, B. Stech and M. Wirbel, Z. Phys. C **34**, 103 (1987).
- [70] A. J. Buras, M. Jamin and M. E. Lautenbacher, Nucl. Phys. B **400**, 75 (1993) hep-ph/9211321.
- [71] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, Phys. Lett. B **301**, 263 (1993) hep-ph/9212203.
- [72] M. Ciuchini, E. Franco, G. Martinelli and L. Reina, Nucl. Phys. B **415**, 403 (1994) hep-ph/9304257.
- [73] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996) hep-ph/9512380.
- [74] E. Barberio *et al.* [Heavy Flavor Averaging Group (HFAG) Collaboration], arXiv:0704.3575 [hep-ex].
- [75] S. J. Lee and M. Neubert, Phys. Rev. D **72**, 094028 (2005) hep-ph/0509350.
- [76] H. Y. Cheng, C. K. Chua and K. C. Yang, Phys. Rev. D **73**, 014017 (2006) hep-ph/0508104.
- [77] H. Y. Cheng, C. K. Chua and K. C. Yang, Phys. Rev. D **77**, 014034 (2008) arXiv:0705.3079[hep-ph].